Electromagnetics

Electromagnetics

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \cdot \boldsymbol{D} = \rho_v$$

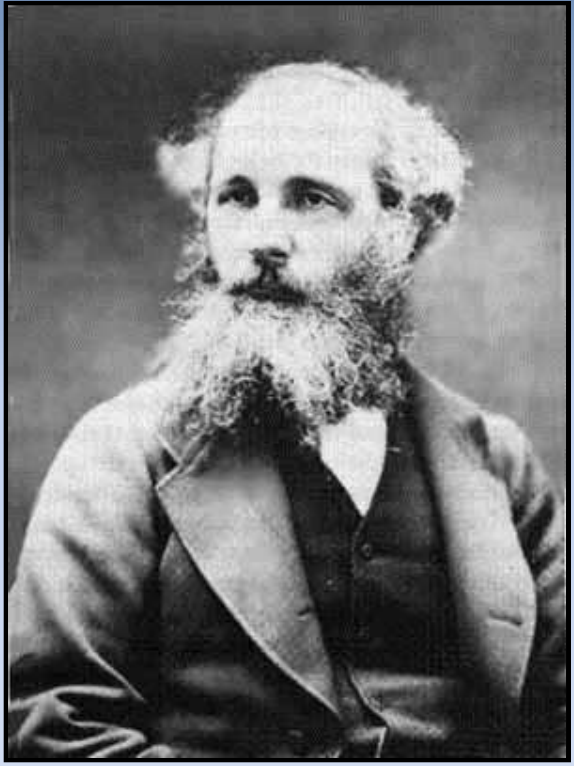
$$\boldsymbol{D} = \epsilon \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

$\boldsymbol{E}$ = electric field strength (volts per meter)
$\boldsymbol{D}$ = electric flux density (coulombs per square meter)
$\boldsymbol{H}$ = magnetic field strength (amperes per meter)
$\boldsymbol{B}$ = magnetic flux density (webers per square meter) or (teslas)

$\boldsymbol{J}$ = electric current density (amperes per square meter)
ρ_v = electric charge density (coulombs per square meter)

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



$$\nabla \cdot \boldsymbol{J} + \frac{\partial \rho_v}{\partial t} = 0$$

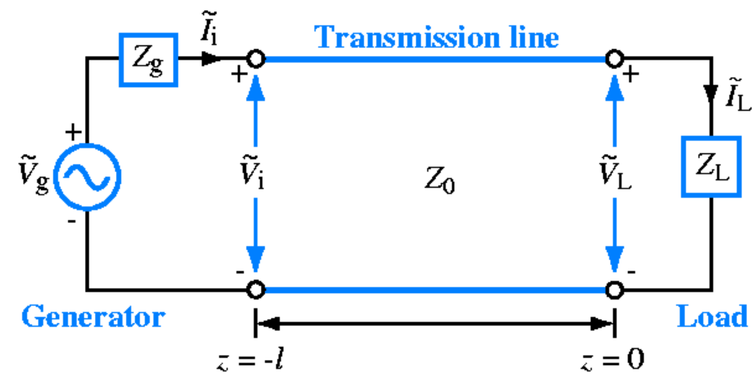
$$\frac{d^2 \tilde{V}(z)}{dz^2} = \gamma^2 \tilde{V}(z)$$

$$\gamma \equiv \sqrt{(R'+j\omega L')(G'+j\omega C')} = \alpha + j\beta$$

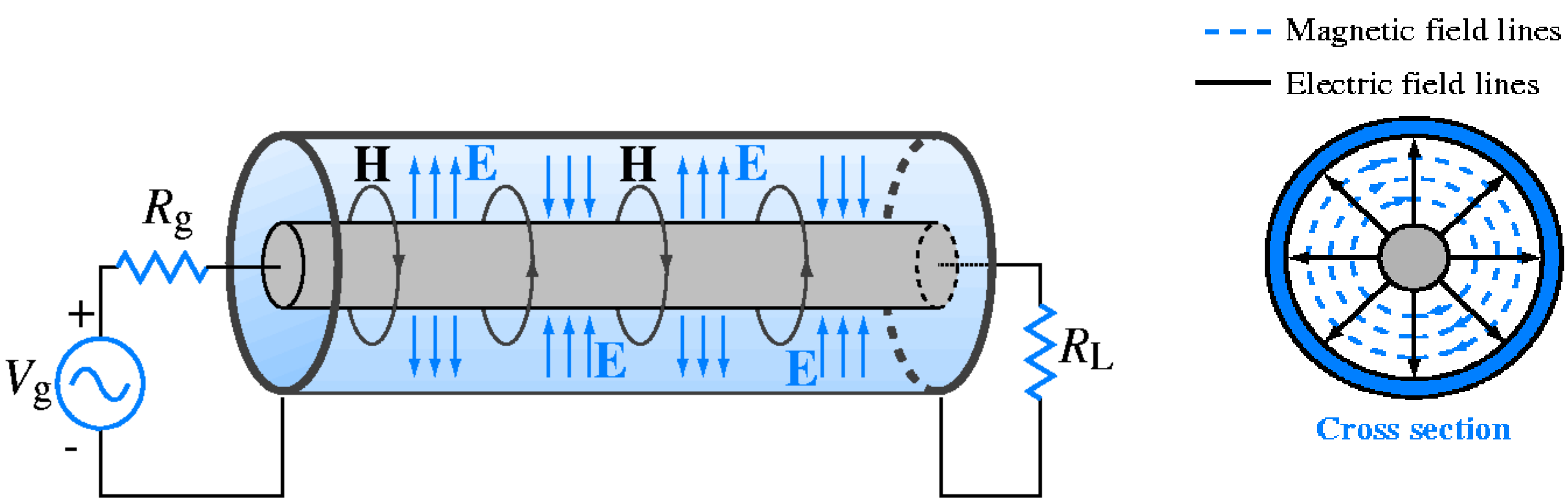
$$\frac{d^2 \tilde{I}(z)}{dz^2} = \gamma^2 \tilde{I}(z)$$

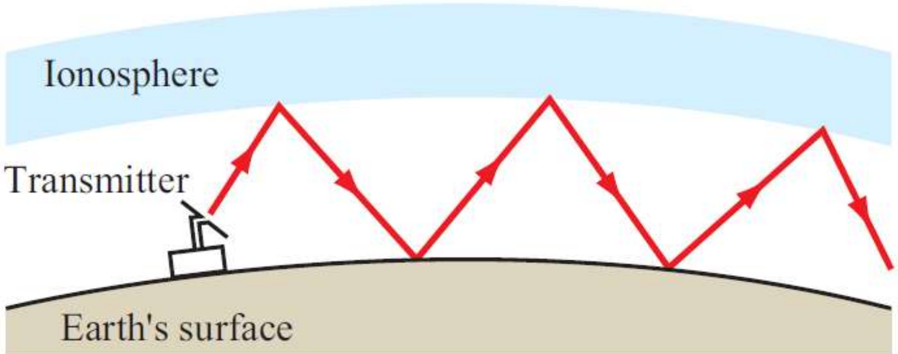
$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

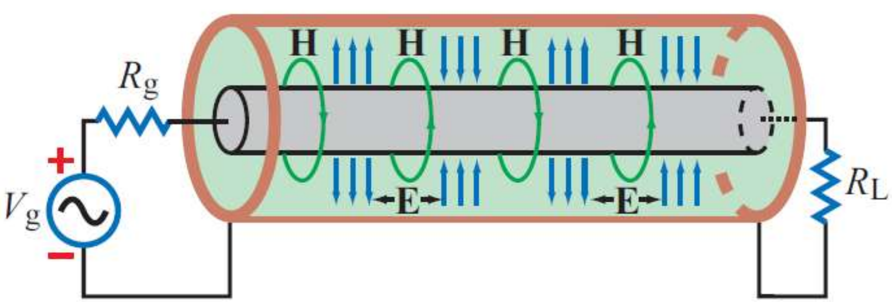
$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$



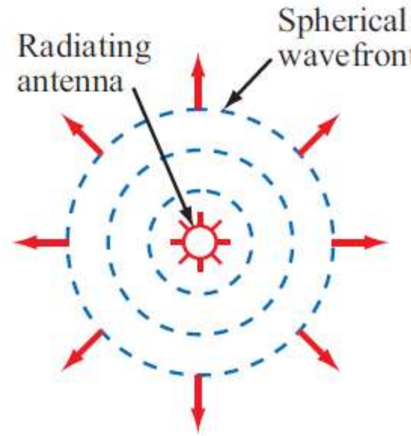
$$\tilde{V}(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} \Rightarrow v(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z)$$



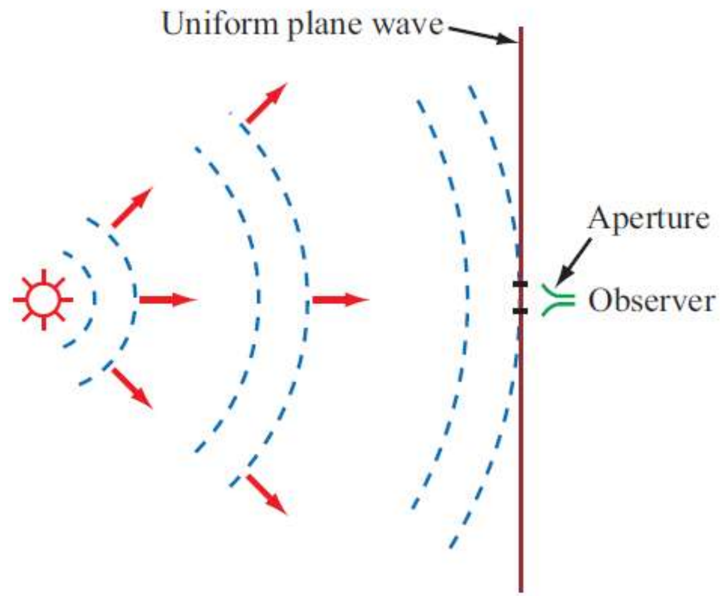




Guided EM Waves



(a) Spherical wave



(b) Plane-wave approximation

Unbounded EM Waves

Remember Last Lecture?

What about free-space ???

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

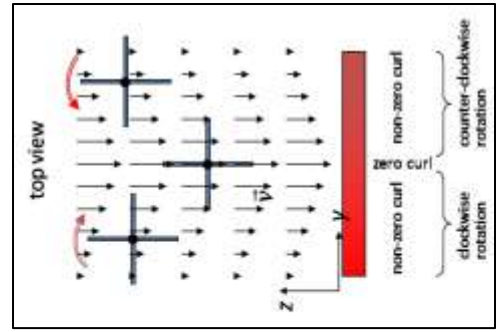
$\mu \vec{H}$ $\quad$ 0 $\quad$ $\epsilon \vec{E}$

$$\Downarrow$$

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

$$\Downarrow$$

$$\nabla \times \nabla \times \vec{E} = -\mu\epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$



Remember Vector Analysis?

Second Derivatives

(9) $\quad \nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\quad \nabla \times (\nabla f) = 0$

(11) $\quad \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$$\Rightarrow \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu\epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$

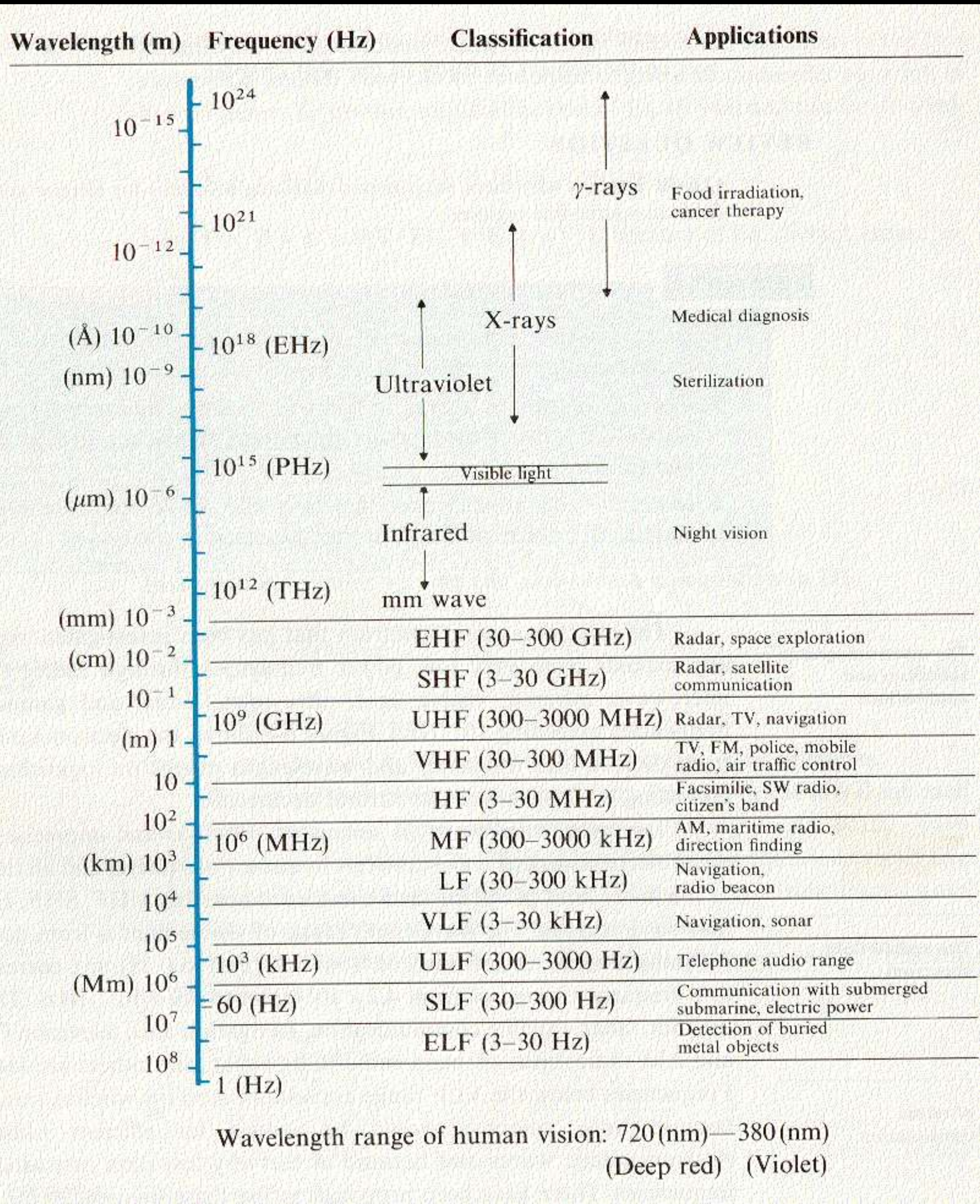
$=0$ in free space...

$$\Rightarrow \nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$

with $\nabla^2 \vec{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{E}$

in Cartesian...

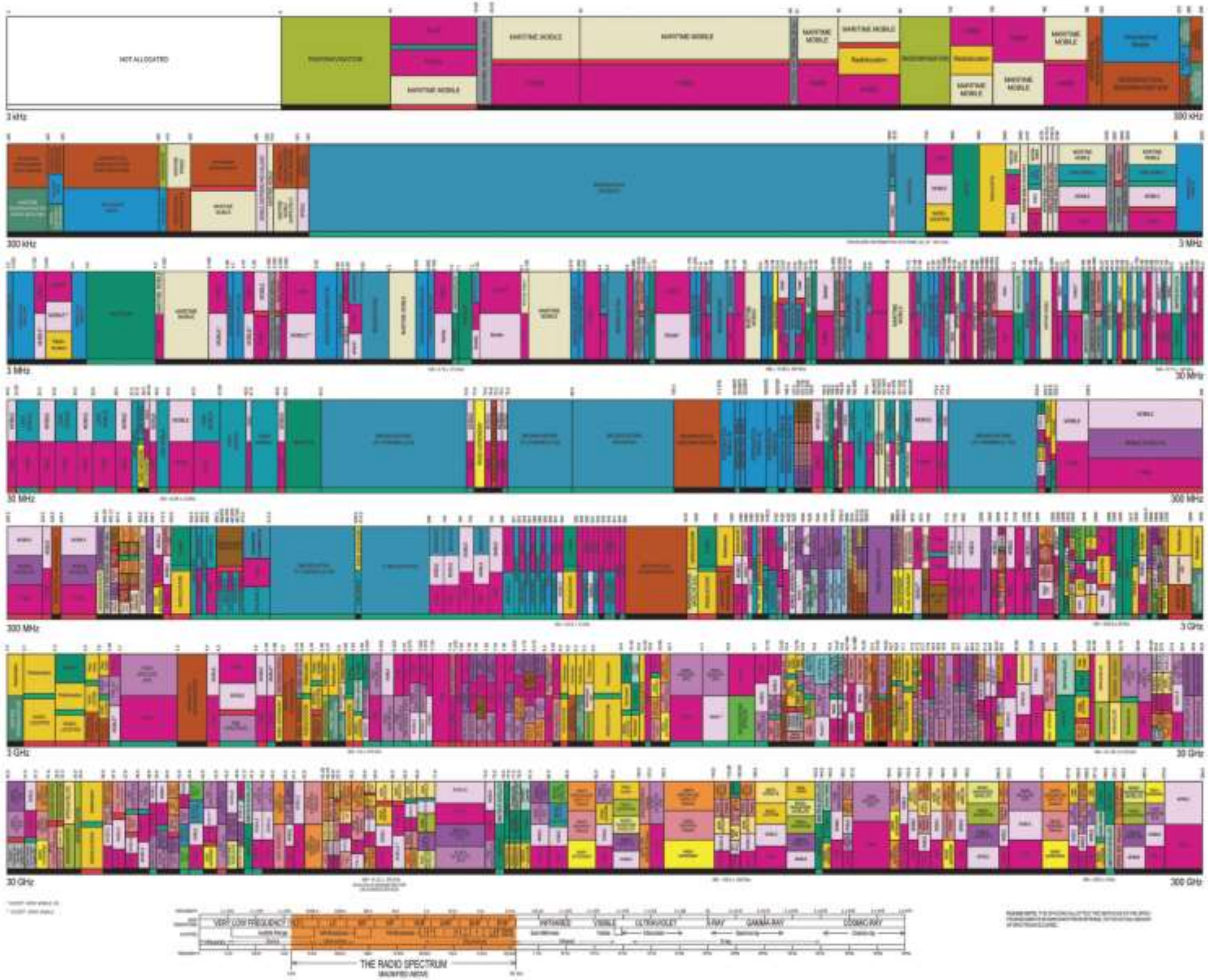
Note: RHS has higher order $\vec{E}$ terms in non-linear media...



What about EE350 ?? ☺

UNITED STATES FREQUENCY ALLOCATIONS

THE RADIO SPECTRUM



RADIO SERVICES COLOR LEGEND

ACTIVITY CODE

ALLOCATION USAGE DESIGNATION

U.S. DEPARTMENT OF COMMERCE
National Telecommunications and Information Administration
Office of Spectrum Management

October 2003

Time-Harmonic Fields

e.g. $\vec{E}(x,y,z,t) = \mathrm{Re}\left[\underbrace{\tilde{E}(x,y,z)}_{\text{Phasors... (like in 210 and in T-lines!)}} e^{j\omega t}\right]$

Note: $\frac{\partial}{\partial t}\vec{E} \Rightarrow j\omega\tilde{E}$

<u>Maxwell in Phasor-land</u>

$$\nabla \cdot \widetilde{\mathbf{E}} = \tilde{\rho}_{\mathrm{V}}/\varepsilon,$$

$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}},$$

$$\nabla \cdot \widetilde{\mathbf{H}} = 0,$$

$$\nabla \times \widetilde{\mathbf{H}} = \widetilde{\mathbf{J}} + j\omega\varepsilon\widetilde{\mathbf{E}},$$

The whole idea of $\varepsilon = \varepsilon_R \varepsilon_0$ is a paramaratization… It only holds true when there are an enormous number of molecules making up a media.

If the material allows flow of charge (i.e. it's conductive) then Ohm's law says:

$$\vec{J} = \sigma \vec{E} \quad \Rightarrow \quad \nabla x \vec{H} = \vec{J} + j\omega \vec{D} = j\omega\varepsilon_c \vec{E}$$

To include this, expand our definition of complex permittivity:

$$\varepsilon_C = \varepsilon_R \varepsilon_0 - j\frac{\sigma}{\omega}$$

Maxwell in (free charge free)
<u>Phasor-land</u>

$$\nabla \cdot \widetilde{\mathbf{E}} = 0,$$
$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu \widetilde{\mathbf{H}},$$
$$\nabla \cdot \widetilde{\mathbf{H}} = 0,$$
$$\nabla \times \widetilde{\mathbf{H}} = j\omega\varepsilon_c \widetilde{\mathbf{E}}.$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$

$$\nabla^2 \tilde{E} = \mu\epsilon (j\omega)^2 \tilde{E}$$

$$\boxed{\nabla^2 \tilde{E} = -\mu\epsilon\omega^2 \tilde{E}}$$

…and analogously for the corresponding magnetic field!!!

Given an electric field

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \sin ay \cos(\omega t - kz),$$

where E_0, a, ω, and k are constants, find $\mathbf{H}$.

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \sin ay \cos(\omega t - kz),$$

Switch to Phasor...

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}} E_0 \sin ay\, e^{-jkz},$$

Faraday

$$\widetilde{\mathbf{H}} = -\frac{1}{j\omega\mu} \nabla \times \widetilde{\mathbf{E}}$$

$$= -\frac{1}{j\omega\mu}\left[\hat{\mathbf{y}} \frac{\partial}{\partial z}(E_0 \sin ay\, e^{-jkz}) - \hat{\mathbf{z}} \frac{\partial}{\partial y}(E_0 \sin ay\, e^{-jkz})\right]$$

$$= \frac{E_0}{\omega\mu}[\hat{\mathbf{y}}\, k \sin ay - \hat{\mathbf{z}}\, ja \cos ay] e^{-jkz},$$

$$\mathbf{H} = \mathfrak{Re}[\widetilde{\mathbf{H}} e^{j\omega t}]$$

and back again!

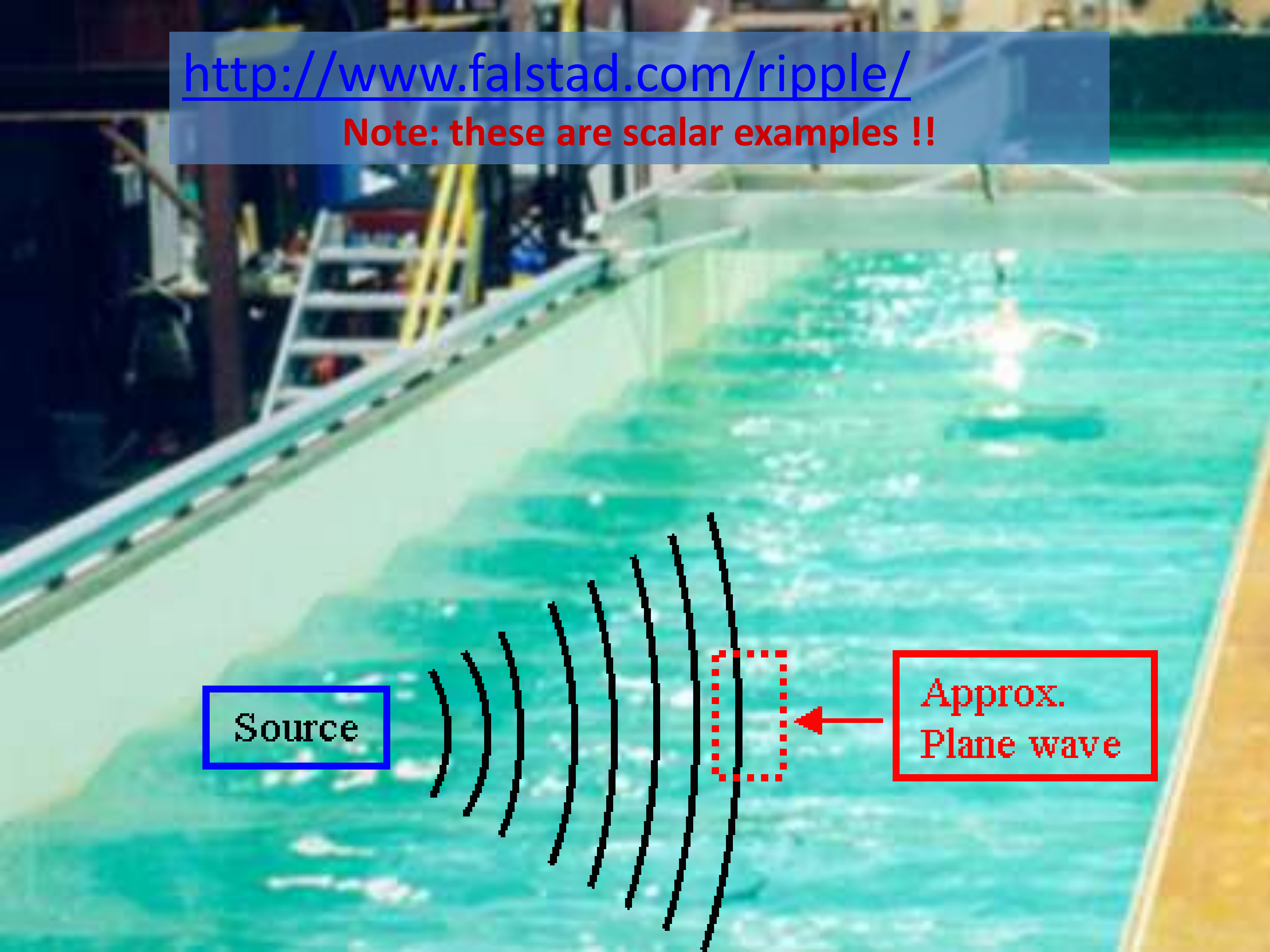
$$= \mathfrak{Re}\left\{\frac{E_0}{\omega\mu}[\hat{\mathbf{y}}\, k \sin ay + \hat{\mathbf{z}}\, a \cos ay\, e^{-j\pi/2}] e^{-jkz} e^{j\omega t}\right\}$$

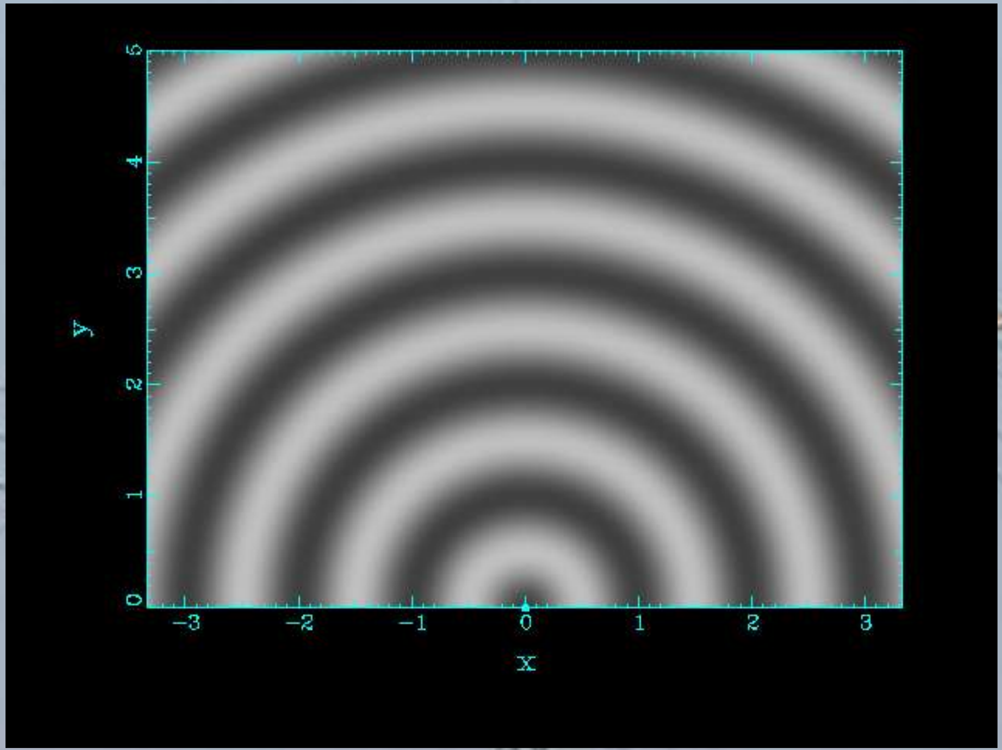
$$= \frac{E_0}{\omega\mu}\left[\hat{\mathbf{y}}\, k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}}\, a \cos ay \cos\left(\omega t - kz - \frac{\pi}{2}\right)\right]$$

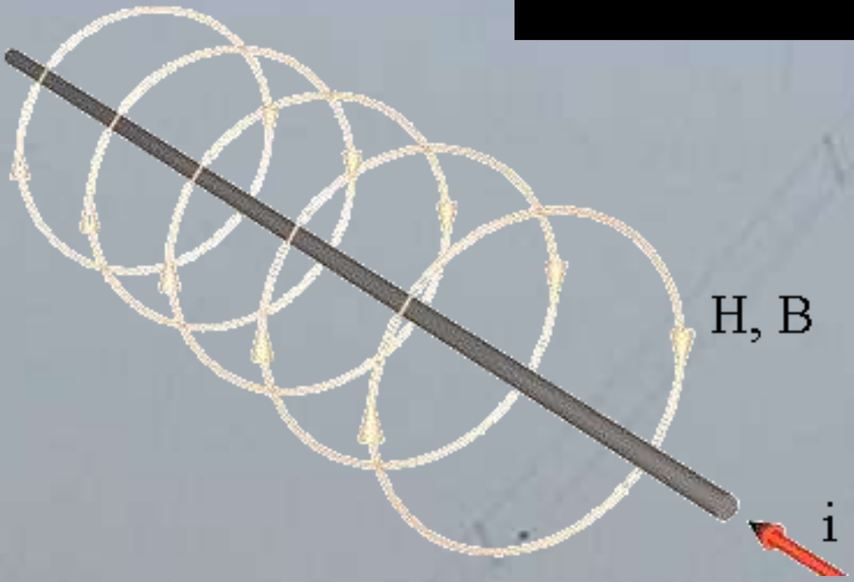
$$= \frac{E_0}{\omega\mu}[\hat{\mathbf{y}}\, k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}}\, a \cos ay \sin(\omega t - kz)].$$

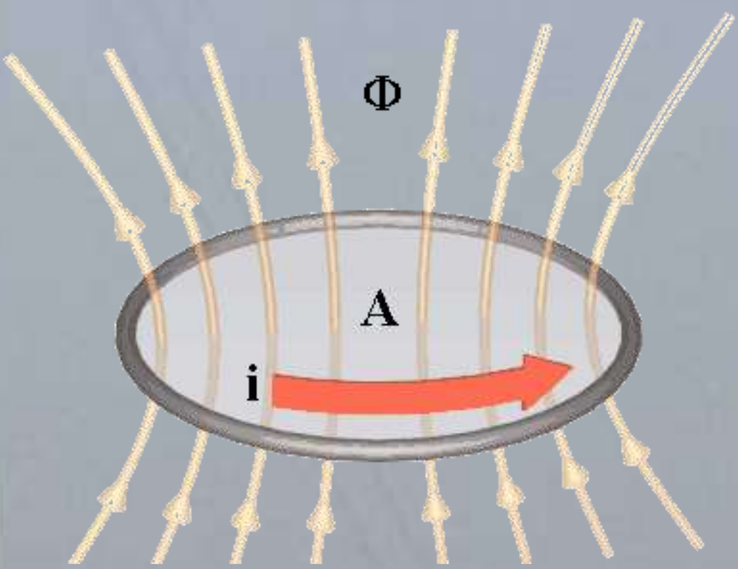
http://www.falstad.com/ripple/

Note: these are scalar examples !!




y
x
0
1
2
3
4
5
−3
−2
−1
0
1
2
3


H, B
i


Φ
A
i

$$\nabla^2 \tilde{E} = \underbrace{-\mu\epsilon\omega^2}_{\gamma^2} \tilde{E}$$

Propagation Constant

$\gamma^2 \rightarrow \gamma = \alpha + j\beta$

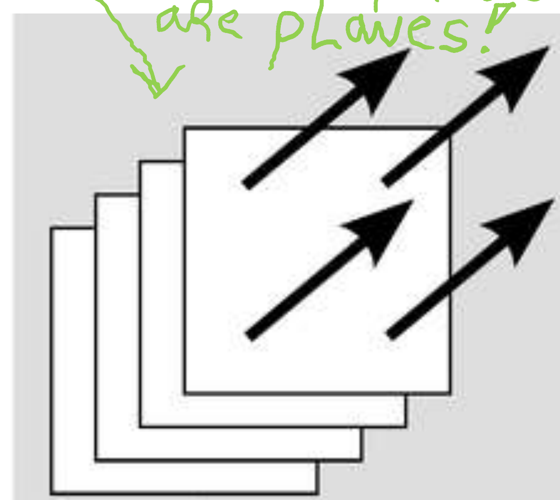
If lossless, $\alpha = 0$, $\beta = k \equiv$ wavenumber $= \omega\sqrt{\mu\epsilon}$

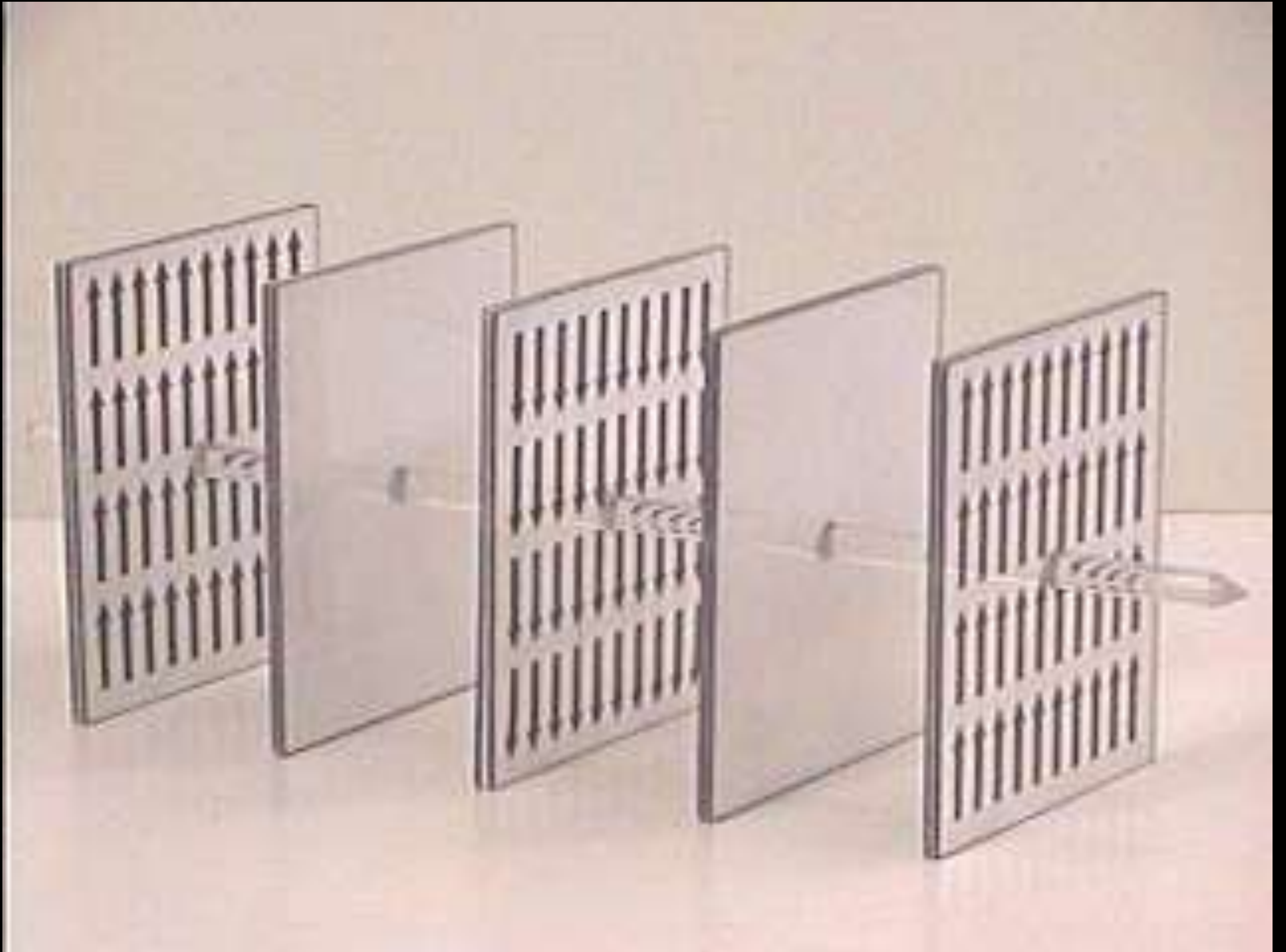
Special Case: Uniform Plane Wave (UPW)

at fixed time, surfaces of constant phase are planes!

$$\vec{E}(x,y,z,t) = E_0 \cos\left[\underbrace{\omega t - kz}_{\text{phase}}\right]\hat{x}$$

one of, well, infinitely many solutions!
(all depends on boundary conditions).



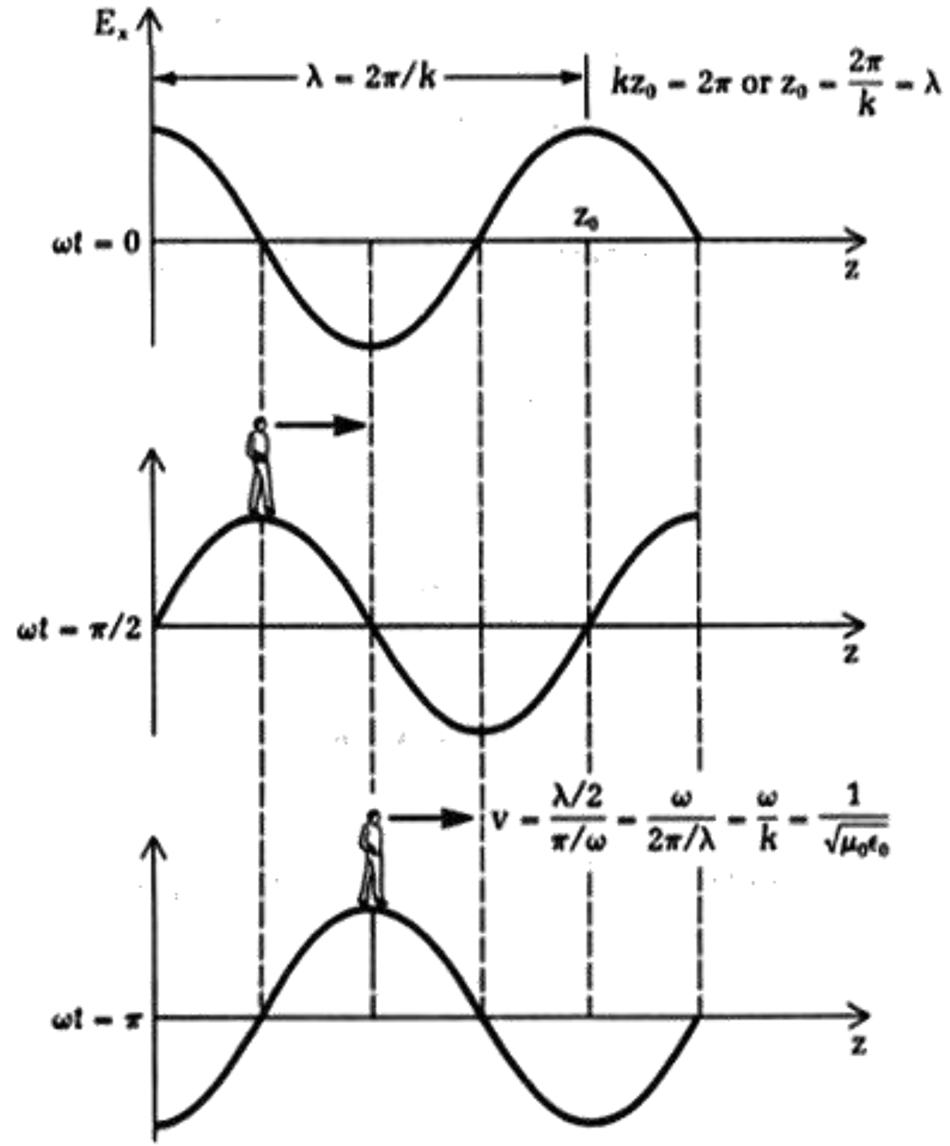


$$\vec{E}(x,y,z,t) = E_0 \cos[\omega t - kz]\hat{x}$$

Point of constant phase….
ωt-kz = constant

$$\Rightarrow \quad \frac{\partial}{\partial t}(\omega t - kz) = \omega - k\frac{\partial z}{\partial t} = 0$$

$$\Rightarrow \quad \text{phase velocity} = v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$



Phase Velocity of UPW

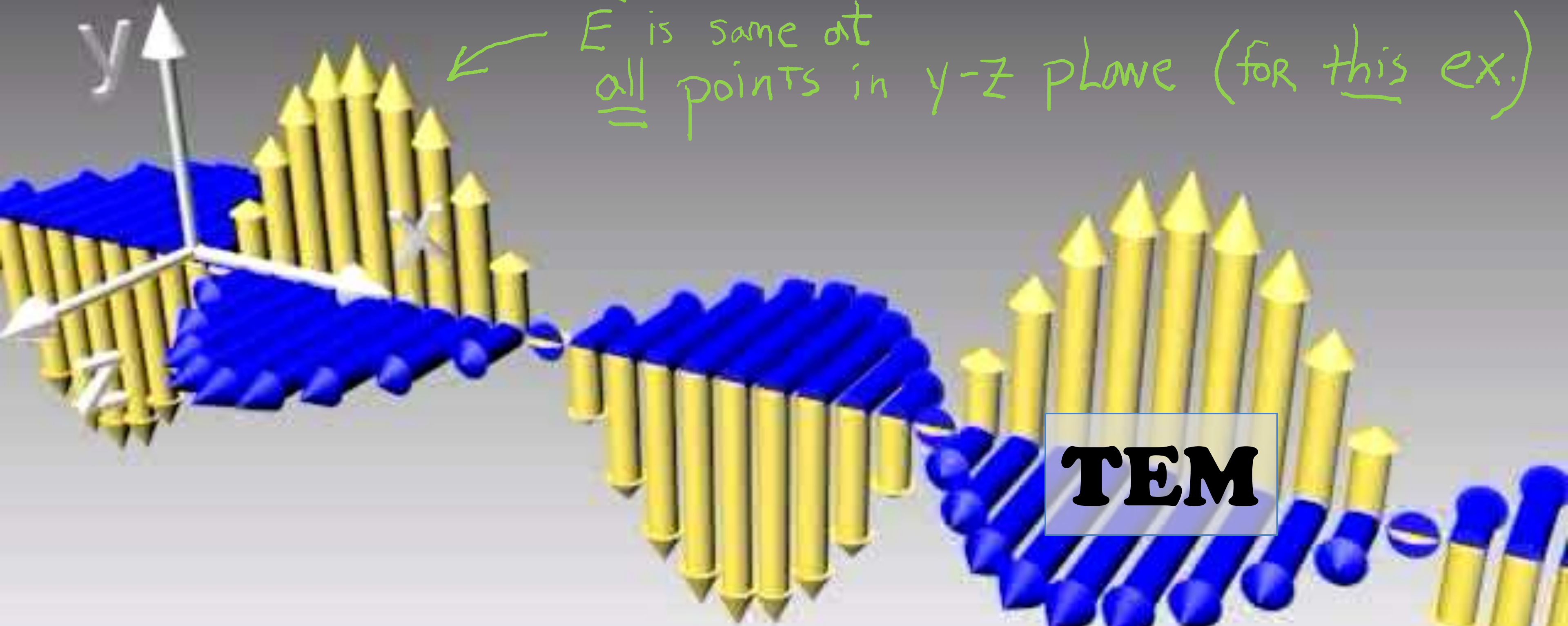
$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

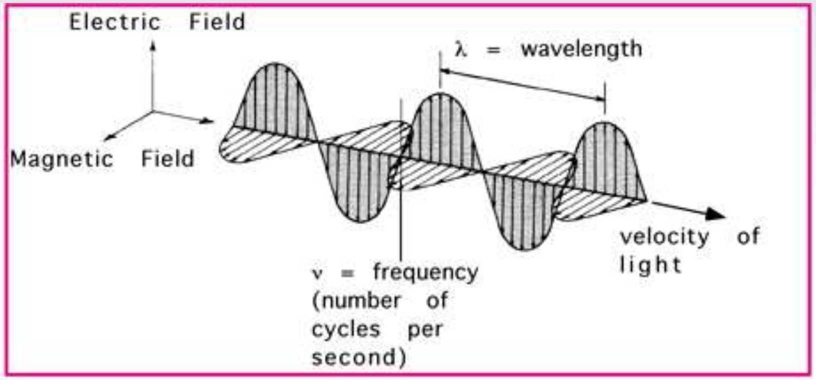
$$\varepsilon_0 \approx 8.854 \times 10^{-12} \text{ F/m} \qquad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\therefore \quad \underline{v \approx 2.998 \times 10^{8} \text{ m/s} = c}$$

$$\vec{E}(x,y,z,t) = E_0 \cos[\omega t - kx]\hat{y}$$

$\vec{E}$ is same at <u>all</u> points in y-z plane (for <u>this</u> ex.)





The <u>frequency</u> of a wave is a characteristic of the source of the wave and does not depend on the medium parameters, as long as the medium is <u>linear</u>.

The frequency does *not* change when the wave travels in different linear media.

$$\lambda = \frac{2\pi}{k} \qquad \lambda f = c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\vec{E}(x,y,z,t) = \vec{E}_0 \cos\left[\omega t - \vec{k} \bullet \vec{r}\right] \quad V/m$$

General UPW propagating in $\vec{k}$ direction.

$\vec{E} \perp \vec{k}$

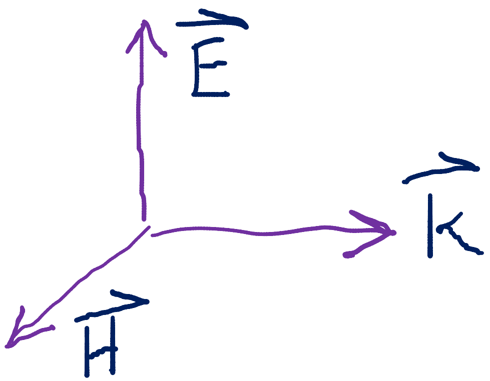
$k\,\hat{k}$

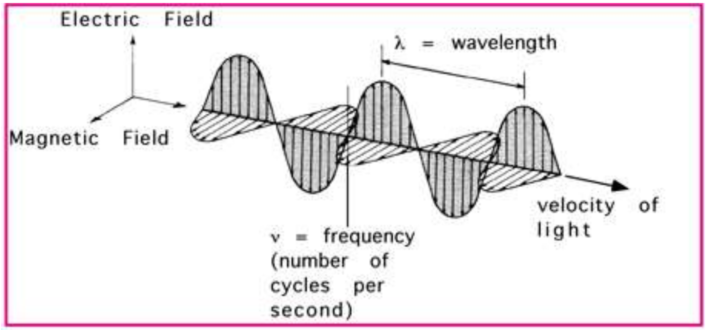
What about that magnetic field??

$$\boxed{\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}},} \xrightarrow[\text{UPWs}]{\text{for}} \begin{cases} \widetilde{H} = \dfrac{1}{\eta}\hat{k} \times \widetilde{E} \\ \widetilde{E} = -\eta\,\hat{k} \times \widetilde{H} \end{cases}$$

with $\eta = \sqrt{\dfrac{\mu}{\epsilon}}$ = intrinsic impedance

Remember Z_0?





$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\,\Omega \approx 120\pi\,\Omega$$

Free Space Impedance

A 10-MHz uniform plane wave is traveling in a nonmagnetic medium with $\mu = \mu_0$ and $\varepsilon_r = 9$. Find (a) the phase velocity, (b) the wavenumber, (c) the wavelength in the medium, and (d) the intrinsic impedance of the medium.

(a)

$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{9}} = 10^8 \quad \text{(m/s)}.$$

(b)

$$k = \frac{\omega}{u_p} = \frac{2\pi \times 10^7}{10^8} = 0.2\pi \quad \text{(rad/m)}.$$

(c)

$$k = \frac{2\pi}{\lambda}, \qquad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10 \quad \text{(m)}.$$

(d)

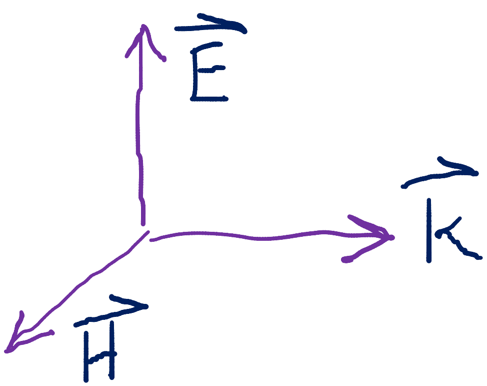
$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \underline{\underline{\sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}}}} = \frac{377}{3} = 125.67\ \Omega.$$

If the magnetic field phasor of a plane wave traveling in a medium with intrinsic impedance $\eta = 100\ \Omega$ is given by $\widetilde{\mathbf{H}} = (\hat{\mathbf{y}}\,10 + \hat{\mathbf{z}}\,20)e^{-j4x}$ (mA/m), find the associated electric field phasor.

$$\widetilde{\mathbf{H}} = (\hat{\mathbf{y}}\,10 + \hat{\mathbf{z}}20)e^{-j4x} \quad \text{(mA/m)}$$

The phase factor of $\widetilde{\mathbf{H}}$ denotes that $\hat{\mathbf{k}} = \hat{\mathbf{x}}$.

$$\begin{aligned}
\widetilde{\mathbf{E}} &= -\eta\hat{\mathbf{k}} \times \widetilde{\mathbf{H}} \\
&= -100[\hat{\mathbf{x}} \times (\hat{\mathbf{y}}\,10 + \hat{\mathbf{z}}\,20)]e^{-j4x} \times 10^{-3} \\
&= (-\hat{\mathbf{z}} + \hat{\mathbf{y}}\,2)e^{-j4x} \quad \text{(V/m)}.
\end{aligned}$$



If the magnetic field phasor of a plane wave traveling in a medium with intrinsic impedance $\eta = 100\ \Omega$ is given by $\widetilde{\mathbf{H}} = \hat{\mathbf{y}}(10e^{-j3x} - 20e^{j3x})$ (mA/m) find the associated electric field phasor.

This magnetic field is composed of two components, one with amplitude of 10 (mA/m) belonging to a wave traveling along $+\hat{\mathbf{x}}$ and another with amplitude of 20 (mA/m) belonging to a separate wave traveling in the opposite direction ($-\hat{\mathbf{x}}$). Hence, we need to treat these two components separately:

$$\widetilde{\mathbf{H}} = \widetilde{\mathbf{H}}_1 + \widetilde{\mathbf{H}}_2$$

Superposition !

with

$$\widetilde{\mathbf{H}}_1 = \hat{\mathbf{y}}\,10e^{-j3x} \quad \text{(mA/m)},$$
$$\widetilde{\mathbf{H}}_2 = -\hat{\mathbf{y}}\,20e^{-j3x} \quad \text{(mA/m)}.$$

For the first wave:

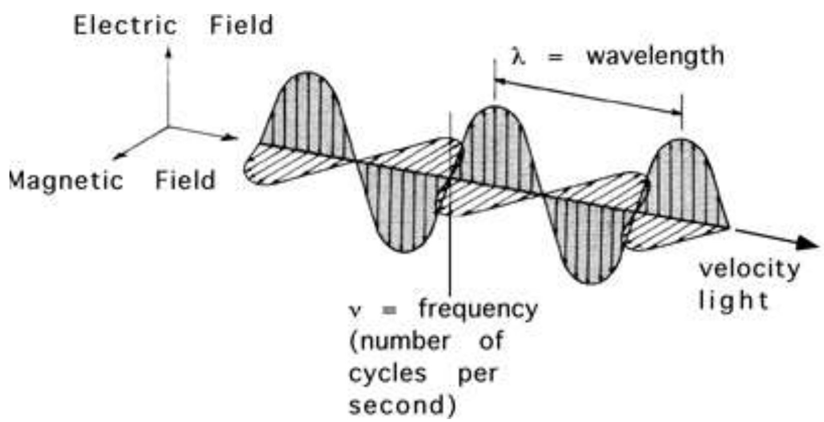
$$\begin{aligned}\widetilde{\mathbf{E}}_1 &= -\eta\hat{\mathbf{k}} \times \widetilde{\mathbf{H}}_1 \\ &= -100(\hat{\mathbf{x}} \times \hat{\mathbf{y}}\,10e^{-j3x}) = -\hat{\mathbf{z}}\,e^{-j3x} \quad \text{(V/m)}.\end{aligned}$$

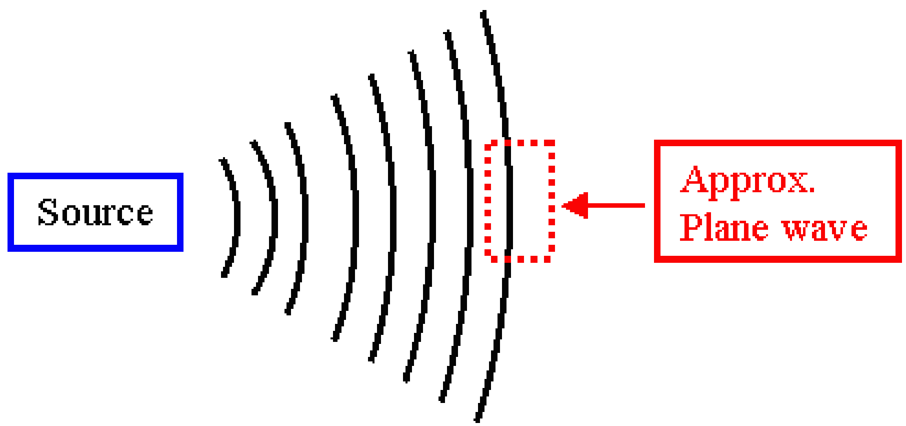
For the second wave:

$$\begin{aligned}\widetilde{\mathbf{E}}_2 &= -100[-\hat{\mathbf{x}} \times (-\hat{\mathbf{y}}\,20e^{j3x})] \\ &= -\hat{\mathbf{z}}\,2e^{j3x} \quad \text{(V/m)}.\end{aligned}$$

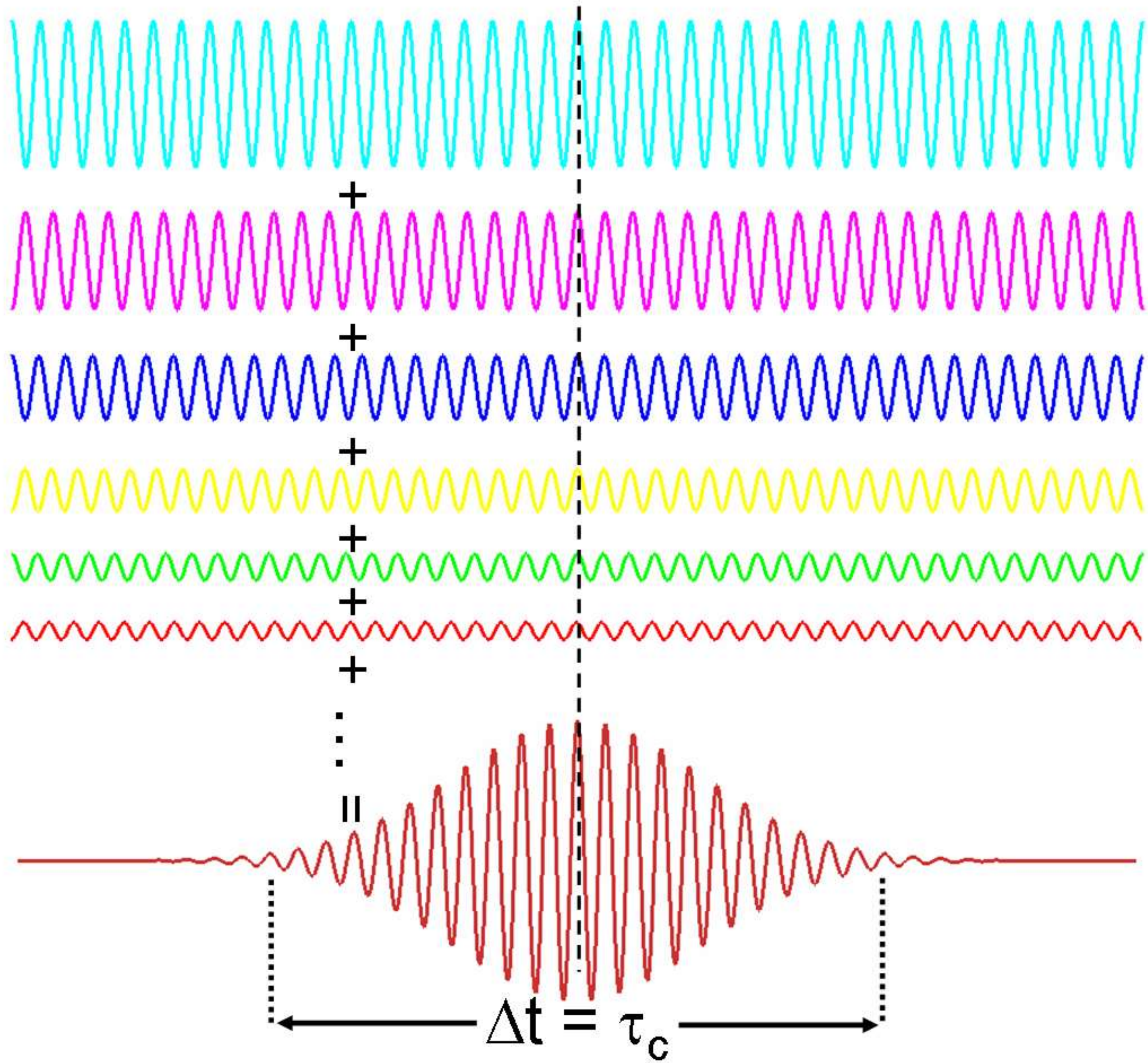
$$\begin{aligned}\widetilde{\mathbf{E}} &= \widetilde{\mathbf{E}}_1 + \widetilde{\mathbf{E}}_2 \\ &= -\hat{\mathbf{z}}(e^{-j3x} + 2e^{j3x}) \quad \text{(V/m)}.\end{aligned}$$

WHY DO WE CARE ABOUT PLANE WAVES?





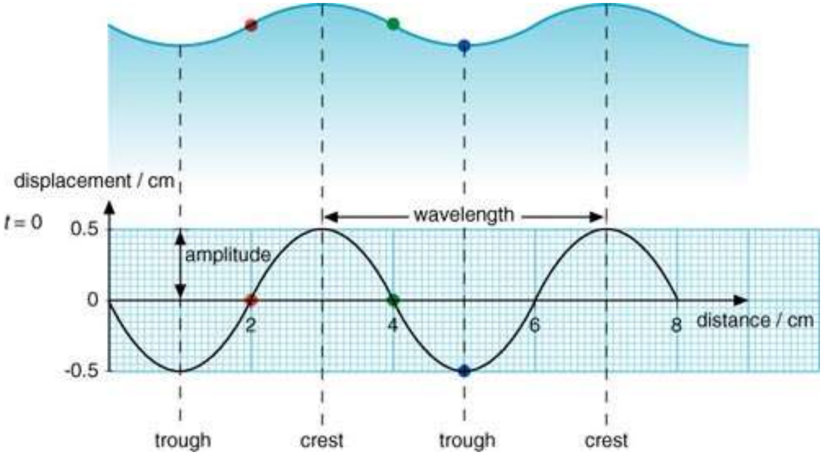
$$t(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T(f_x,f_y)\,e^{+j2\pi(f_x x+f_y y)}\,df_x\,df_y$$

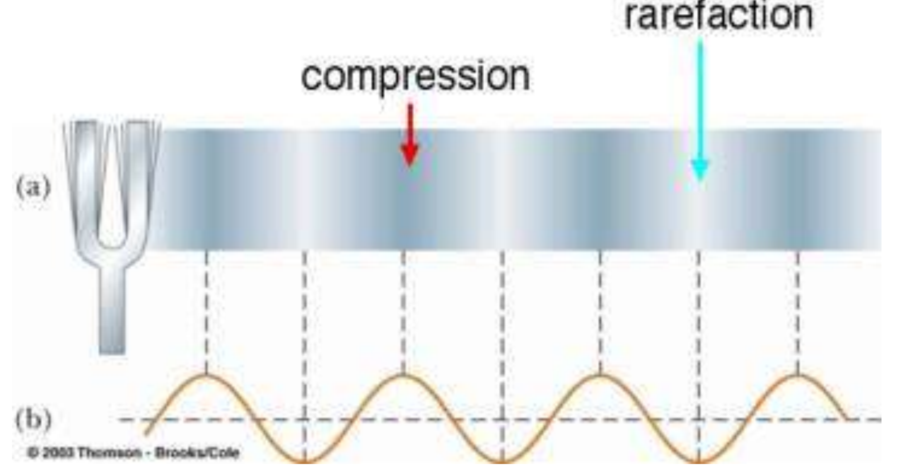

+
+
+
+
+
+
=
$\Delta t = \tau_c$

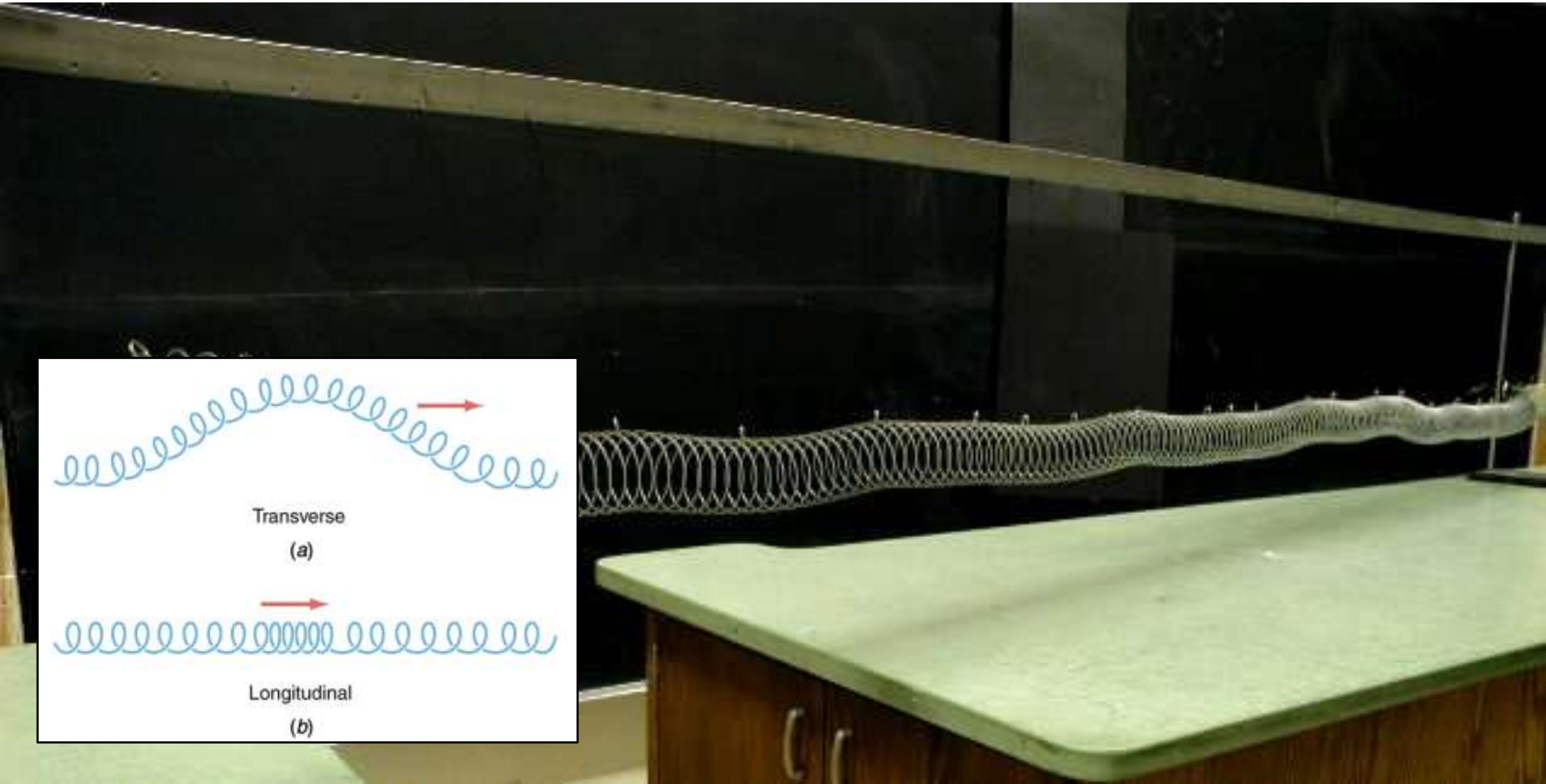
Ranges of relative permittivity??

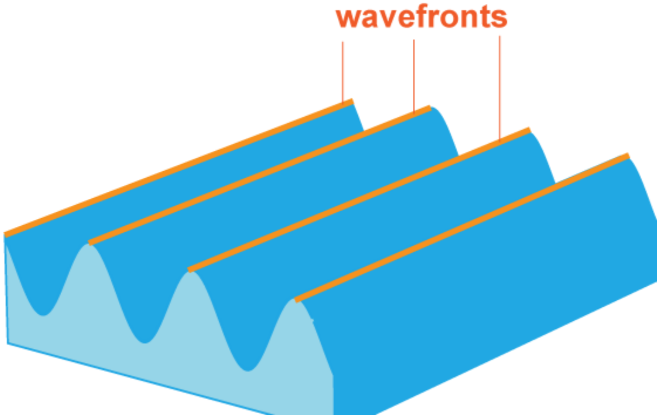
http://en.wikipedia.org/wiki/Dielectric_constant

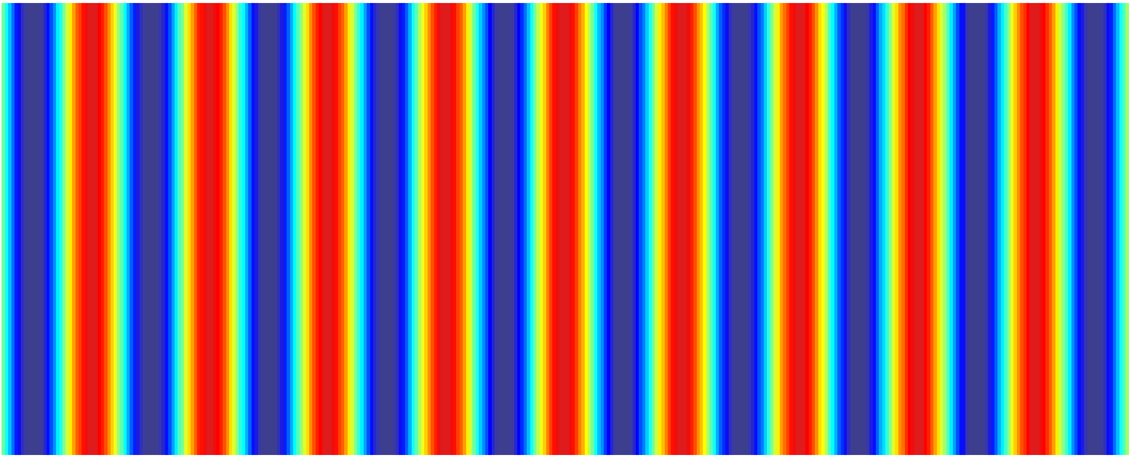
Note: frequency dependent!


displacement / cm
t = 0
0.5
0
-0.5
amplitude
wavelength
2
4
6
8
distance / cm
trough
crest
trough
crest


rarefaction
compression
(a)
(b)
© 2003 Thomson - Brooks/Cole

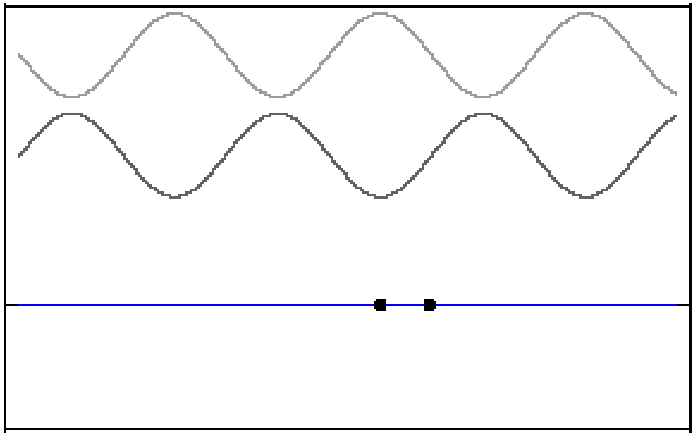

Transverse
(a)
Longitudinal
(b)

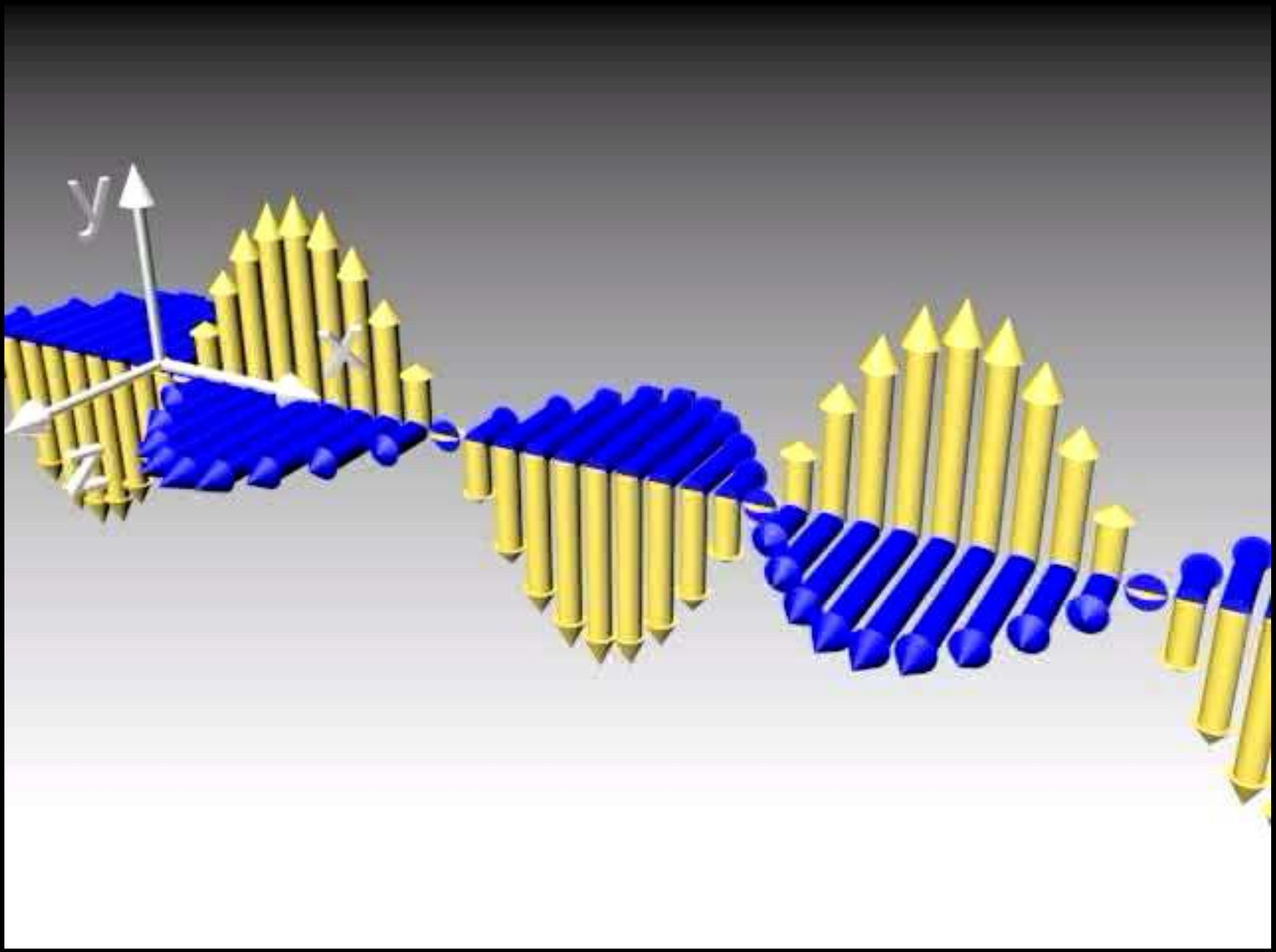




$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

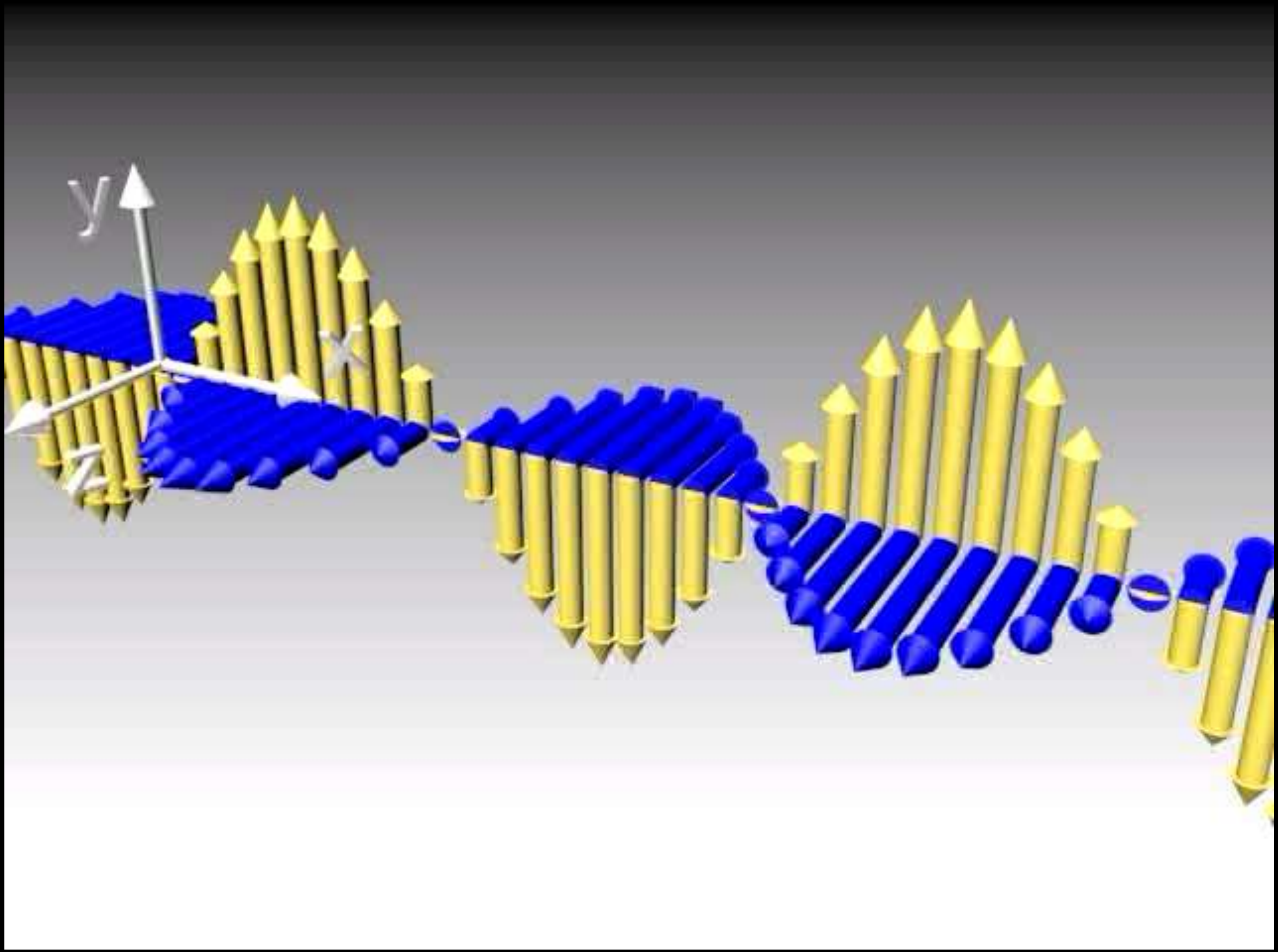
$$\vec{E}(x,y,z,t) = E_0 \cos\left[\omega t - kx\right]\hat{y}$$





Are E and H –fields always in phase???

$\vec{H} = \frac{1}{\eta}\hat{k}\times\vec{E}$ with $\eta \equiv \sqrt{\frac{\mu}{\epsilon}}$ and $\epsilon = \epsilon_R\epsilon_0 - j\frac{\sigma}{\omega}$



Where's the <u>curl</u> in the E-field ???

$$\nabla \times \tilde{E} = -j\omega\mu\tilde{H}$$

Wave decomposition

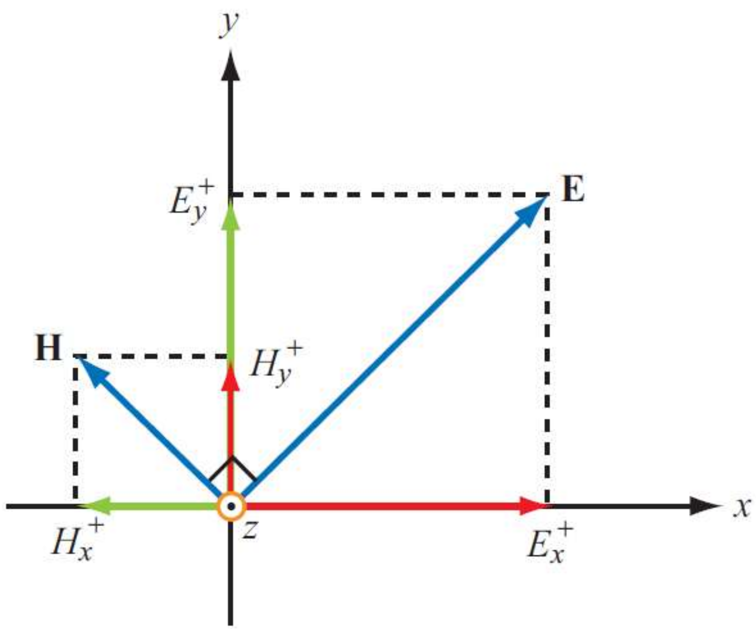


Figure 7-6: The wave (**E**, **H**) is equivalent to the sum of tw waves, one with fields (E_x^+, H_y^+) and another with (E_y^+, H_x^+ with both traveling in the $+z$-direction.

In general, a uniform plane wave traveling in the $+z$-direction may have both x- and y-components, in which case $\widetilde{\mathbf{E}}$ is given by

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}}\,\widetilde{E}_x^+(z) + \hat{\mathbf{y}}\,\widetilde{E}_y^+(z), \tag{7.43a}$$

and the associated magnetic field is

$$\widetilde{\mathbf{H}} = \hat{\mathbf{x}}\,\widetilde{H}_x^+(z) + \hat{\mathbf{y}}\,\widetilde{H}_y^+(z). \tag{7.43b}$$

Application of Eq. (7.39a) gives

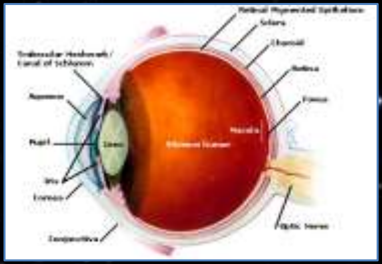
$$\widetilde{\mathbf{H}} = \frac{1}{\eta}\,\hat{\mathbf{z}} \times \widetilde{\mathbf{E}} = -\hat{\mathbf{x}}\,\frac{\widetilde{E}_y^+(z)}{\eta} + \hat{\mathbf{y}}\,\frac{\widetilde{E}_x^+(z)}{\eta}. \tag{7.44}$$

By equating Eq. (7.43b) to Eq. (7.44), we have

$$\widetilde{H}_x^+(z) = -\frac{\widetilde{E}_y^+(z)}{\eta}, \qquad \widetilde{H}_y^+(z) = \frac{\widetilde{E}_x^+(z)}{\eta}. \tag{7.45}$$

A UPW with a wavelength of 532 nm propagates in the +z direction and you are facing in the –z direction…

… what do you see?



$$\vec{E} = \hat{x}\, a \cos(\omega t - kz + \phi_a) + \hat{y}\, b \cos(\omega t - kz + \phi_b)$$

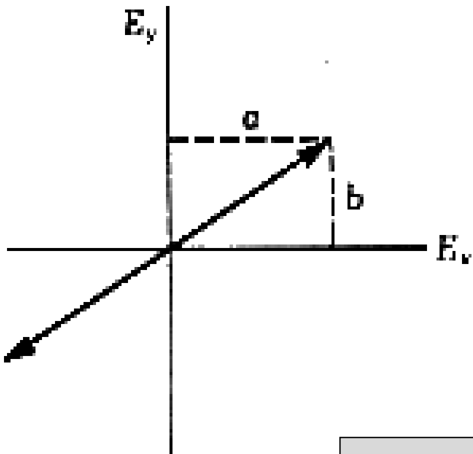
Polarization

$$\tilde{E} = \left[a e^{-j\phi_a}\,\hat{x} + b e^{-j\phi_b}\,\hat{y}\right] e^{-jkz}$$

Why the heck am I using slightly different notation than the book???

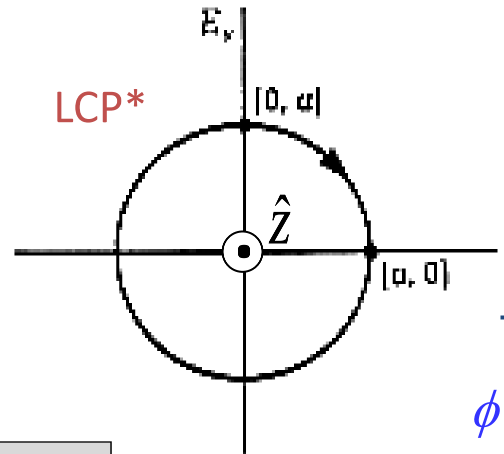
$$\phi = \phi_b - \phi_a$$

Phase *difference*!!!



(a)

LCP*



(b)

Propagation in +z direction

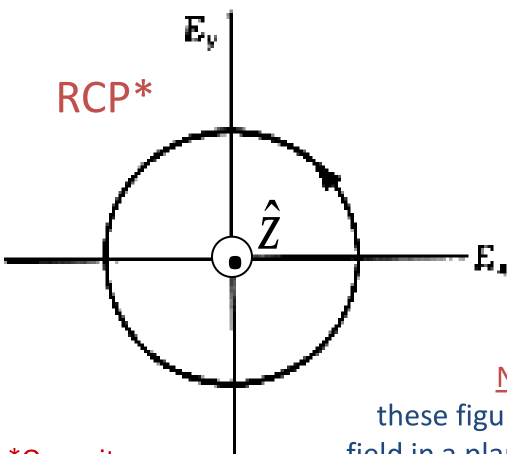
Linear | Circular | Elliptical

Linear: $\phi = 0 \text{ or } \pi$

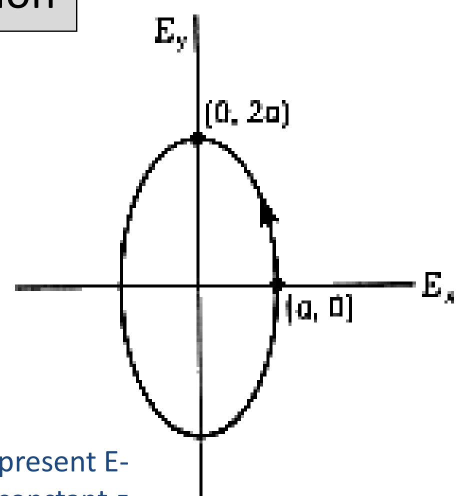
Circular: $\phi = \pm\frac{\pi}{2}$ and $a = b$

Elliptical: $\phi = \text{const.}$, $\frac{a}{b} = \text{const.}$

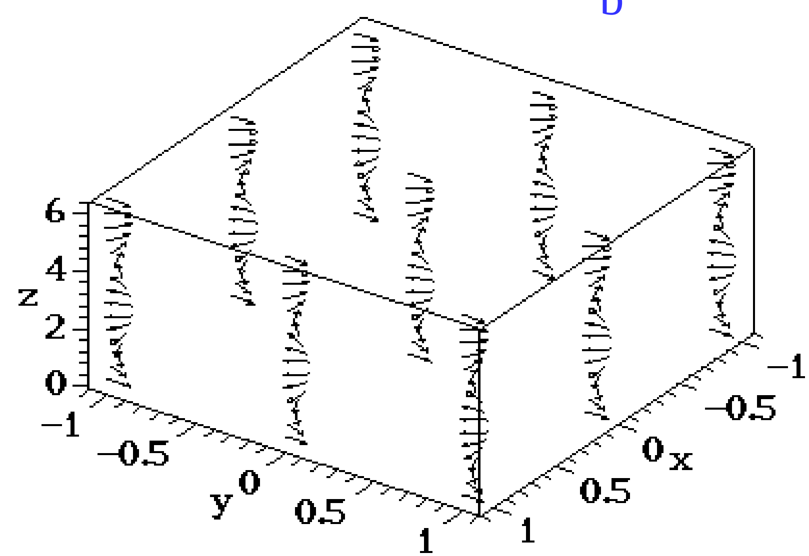
RCP*



(c)



(d)

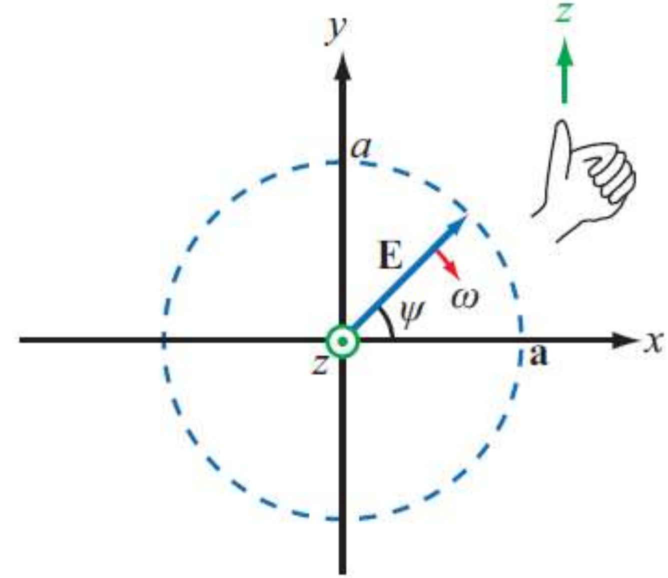


Note:
these figures represent E-field in a plane of constant z, with t varying

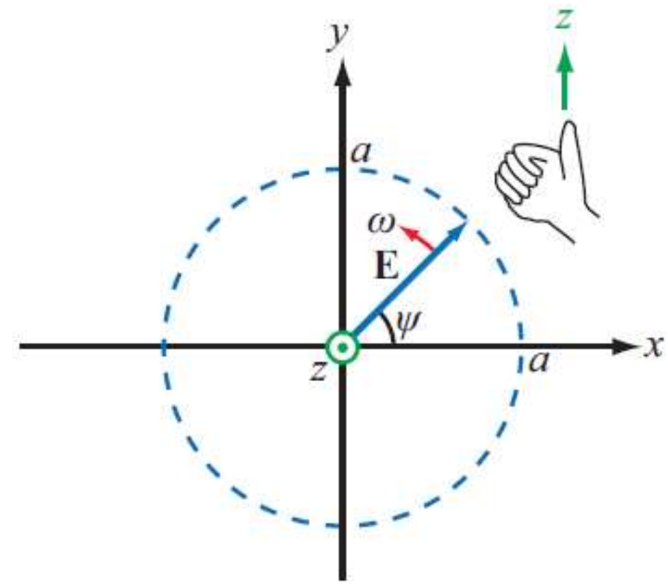
*Opposite for Optics

Polarization Handedness

Polarization handedness is defined in terms of the rotation of **E** *as a function of time in a fixed plane orthogonal to the direction of propagation, which is opposite of the direction of rotation of* **E** *as a function of distance at a fixed point in time.*



(a) LHC polarization



(b) RHC polarization

The electric field of a plane wave is given by

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\,3\cos(\omega t - kz) + \hat{\mathbf{y}}\,4\cos(\omega t - kz) \quad \text{(V/m)}.$$

Determine (a) the polarization state, (b) the modulus of $\mathbf{E}$, and (c) the inclination angle.

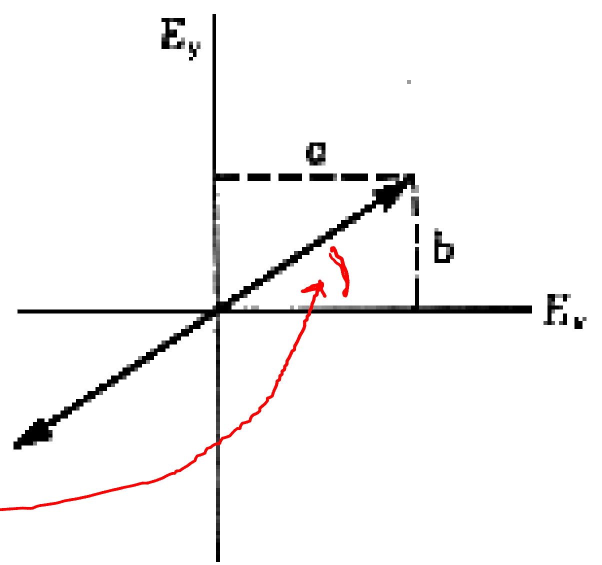
Solution:

(a) Since the x- and y-components are exactly in phase with each other, the polarization state has to be linear. This can also be ascertained by verifying that $\chi = 0$. From the given expression for $\mathbf{E}(z,t)$, $\delta_x = \delta_y = 0$. Hence, $\delta = \delta_y - \delta_x = 0$.

$$\psi_0 = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\frac{4}{3} = 53.1^\circ.$$

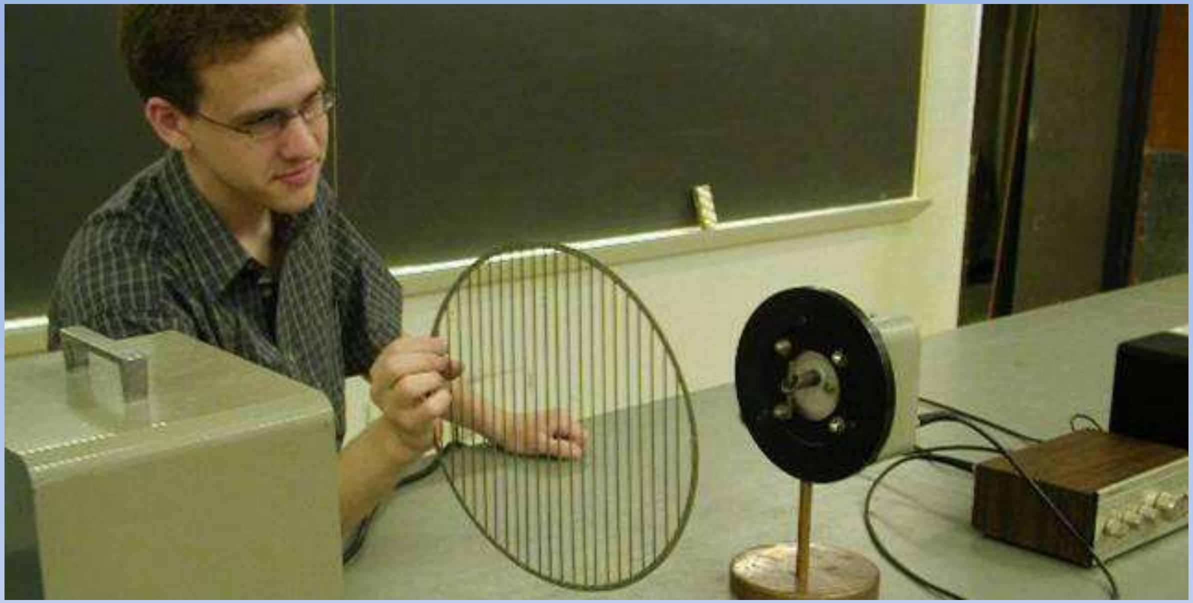
$$\sin 2\chi = (\sin 2\psi_0)\sin\delta = 0$$

$$\chi = 0.$$

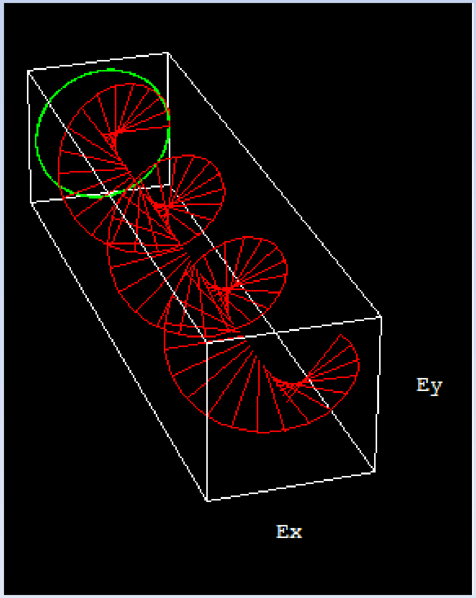


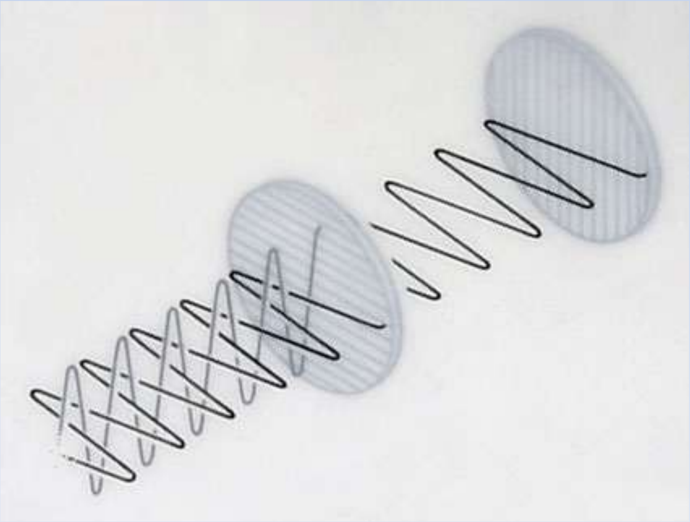
(b) $|\mathbf{E}| = [E_x^2 + E_y^2]^{1/2} = 5\cos(\omega t - kz)$ (V/m).

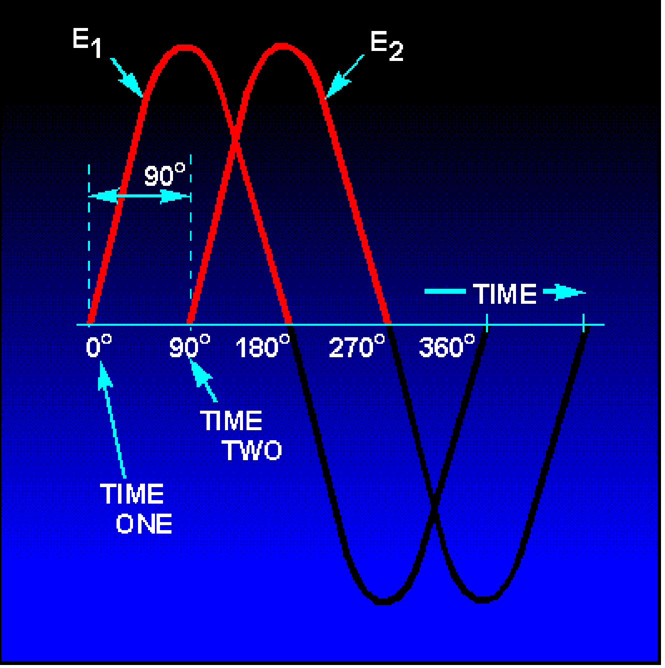
(c) From part (a), $\psi_0 = 53.1^\circ$.

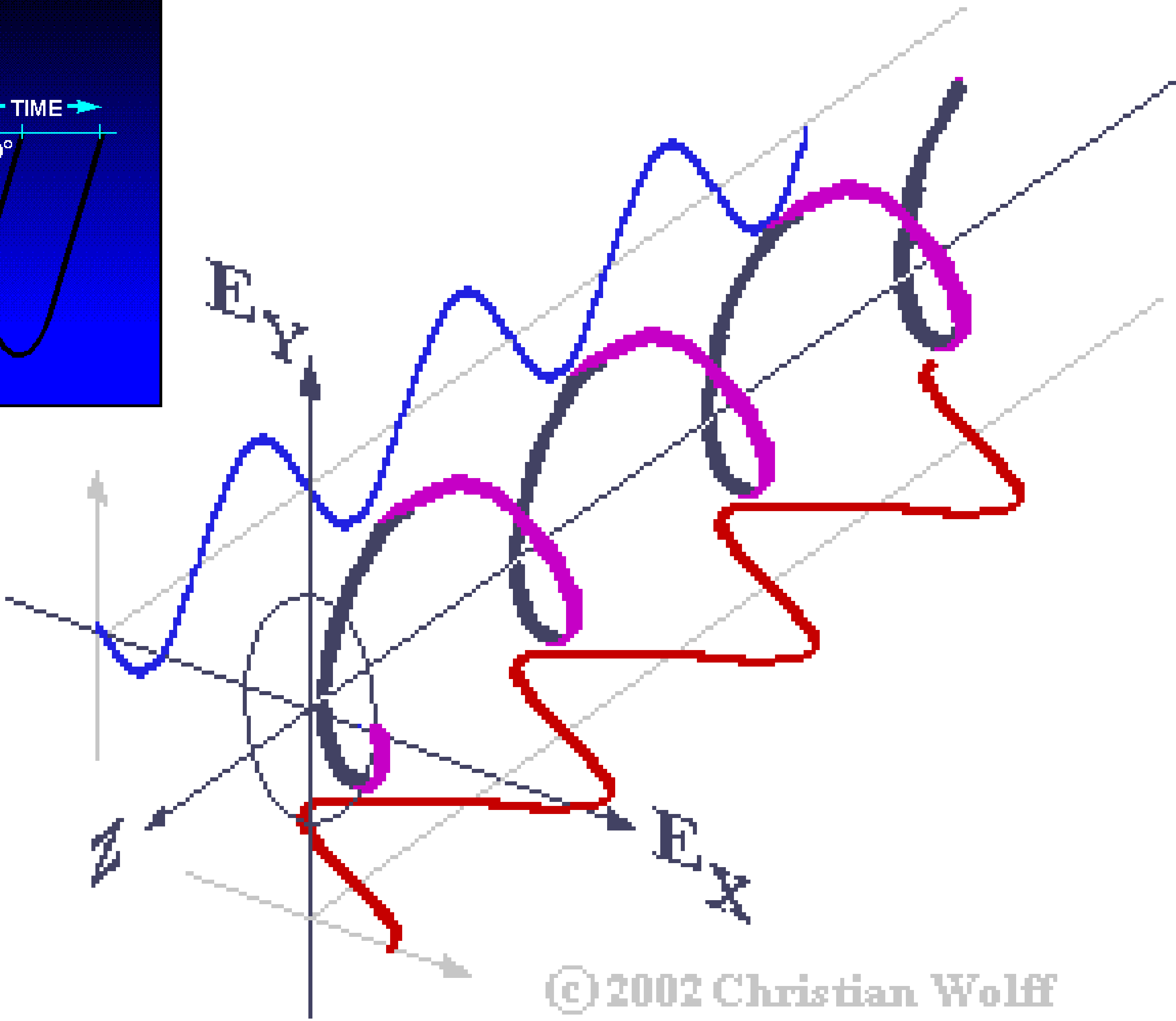


http://webphysics.davidson.edu/Applets/EMWave/EMWave.html





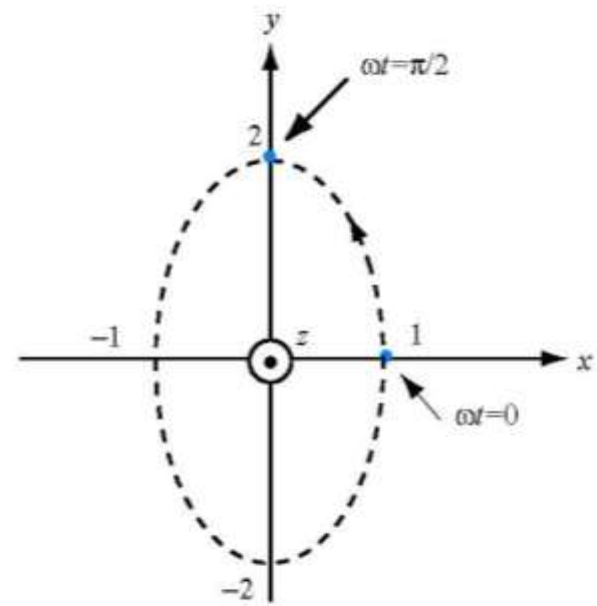

E1
E2
90°
TIME
0°
90°
180°
270°
360°
TIME TWO
TIME ONE


Ey
Ex
z
(c) 2002 Christian Wolff

Plot the locus of $\mathbf{E}(0,t)$ for a plane wave with

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\sin(\omega t + kz) + \hat{\mathbf{y}}\,2\cos(\omega t + kz).$$

Determine the polarization state from your plot.



Locus of E versus time.

Solution:

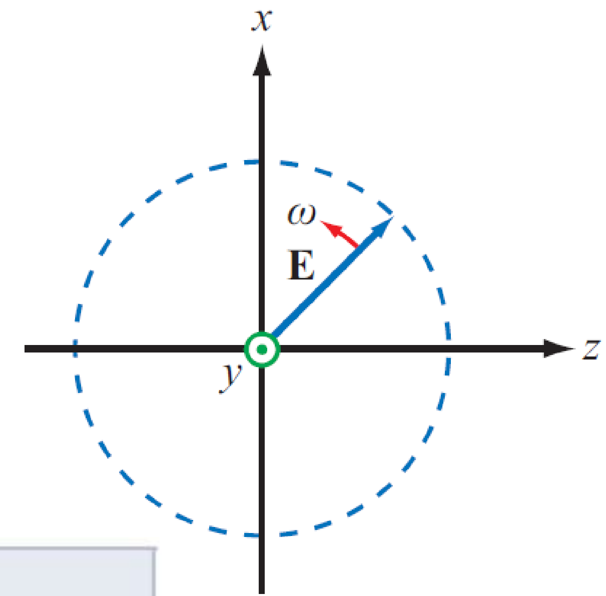
$$\mathbf{E} = \hat{\mathbf{x}}\sin(\omega t + kz) + \hat{\mathbf{y}}\,2\cos(\omega t + kz).$$

Wave direction is $-\hat{\mathbf{z}}$. At $z = 0$,

$$\mathbf{E} = \hat{\mathbf{x}}\sin\omega t + \hat{\mathbf{y}}\,2\cos\omega t.$$

Tip of **E** rotates in accordance with right hand (with thumb pointing along $-\hat{\mathbf{z}}$). Hence, wave state is RHE.


ESCORT



Example 7-2: RHC Polarized Wave

An RHC polarized plane wave with electric field magnitude of 3 (mV/m) is traveling in the $+y$-direction in a dielectric medium with $\varepsilon = 4\varepsilon_0$, $\mu = \mu_0$, and $\sigma = 0$. If the frequency is 100 MHz, obtain expressions for $\mathbf{E}(y, t)$ and $\mathbf{H}(y, t)$.

Solution: Since the wave is traveling in the $+y$-direction, its field must have components along the x- and z-directions. The rotation of $\mathbf{E}(y, t)$ is depicted in Fig. 7-10, where $\hat{\mathbf{y}}$ is out of the page. By comparison with the RHC polarized wave shown in Fig. 7-8(b), we assign the z-component of $\widetilde{\mathbf{E}}(y)$ a phase angle of zero and the x-component a phase shift of $\delta = -\pi/2$.

Cont.

Example 7-2 cont.

Wave with electric field magnitude of 3 (mV/m) traveling in the +*y*-*direction*

$$\widetilde{\mathbf{E}}(y) = \hat{\mathbf{x}}\widetilde{E}_x + \hat{\mathbf{z}}\widetilde{E}_z = \hat{\mathbf{x}}ae^{-j\pi/2}e^{-jky} + \hat{\mathbf{z}}ae^{-jky} = (-\hat{\mathbf{x}}j + \hat{\mathbf{z}})3e^{-jky} \qquad \text{(mV/m)},$$

With $\omega = 2\pi f = 2\pi \times 10^8$ (rad/s), the wavenumber k is

$$k = \frac{\omega\sqrt{\varepsilon_r}}{c} = \frac{2\pi \times 10^8\sqrt{4}}{3 \times 10^8} = \frac{4}{3}\pi \qquad \text{(rad/m)},$$

and application of (7.39a) gives

$$\widetilde{\mathbf{H}}(y) = \frac{1}{\eta}\hat{\mathbf{y}} \times \widetilde{\mathbf{E}}(y) = \frac{1}{\eta}\hat{\mathbf{y}} \times (-\hat{\mathbf{x}}j + \hat{\mathbf{z}})3e^{-jky} = \frac{3}{\eta}(\hat{\mathbf{z}}j + \hat{\mathbf{x}})e^{-jky} \qquad \text{(mA/m)}.$$

Cont.

Example 7-2 cont.

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} \simeq \frac{120\pi}{\sqrt{4}} = 60\pi \qquad (\Omega).$$

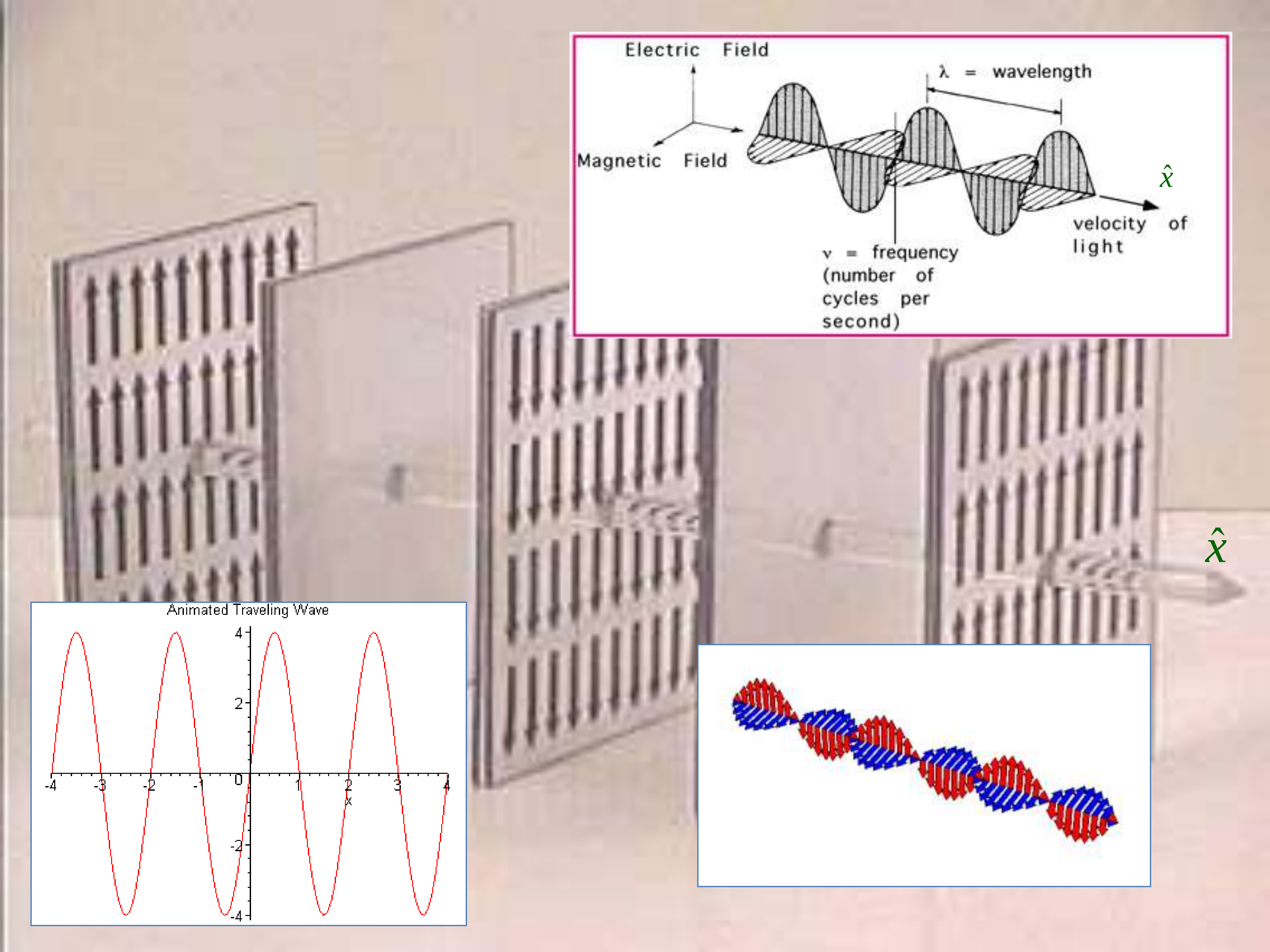
The instantaneous fields $\mathbf{E}(y,t)$ and $\mathbf{H}(y,t)$ are

$$\begin{aligned}\mathbf{E}(y,t) &= \mathfrak{Re}\left[\widetilde{\mathbf{E}}(y)\, e^{j\omega t}\right] \\ &= \mathfrak{Re}\left[(-\hat{\mathbf{x}}j + \hat{\mathbf{z}})3e^{-jky}e^{j\omega t}\right] \\ &= 3[\hat{\mathbf{x}}\sin(\omega t - ky) + \hat{\mathbf{z}}\cos(\omega t - ky)] \qquad (\text{mV/m})\end{aligned}$$

and

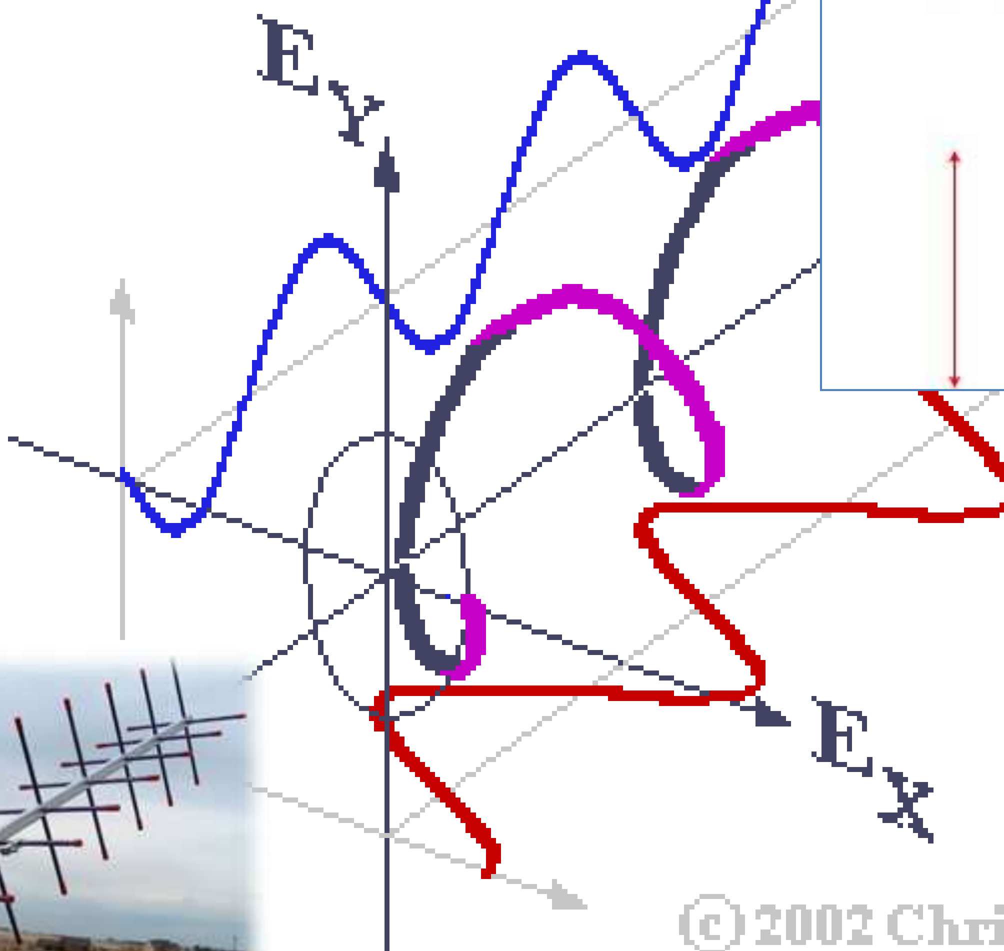
$$\begin{aligned}\mathbf{H}(y,t) &= \mathfrak{Re}\left[\widetilde{\mathbf{H}}(y)\, e^{j\omega t}\right] \\ &= \mathfrak{Re}\left[\frac{3}{\eta}(\hat{\mathbf{z}}j + \hat{\mathbf{x}})e^{-jky}e^{j\omega t}\right] \\ &= \frac{1}{20\pi}[\hat{\mathbf{x}}\cos(\omega t - ky) - \hat{\mathbf{z}}\sin(\omega t - ky)] \; (\text{mA/m}).\end{aligned}$$

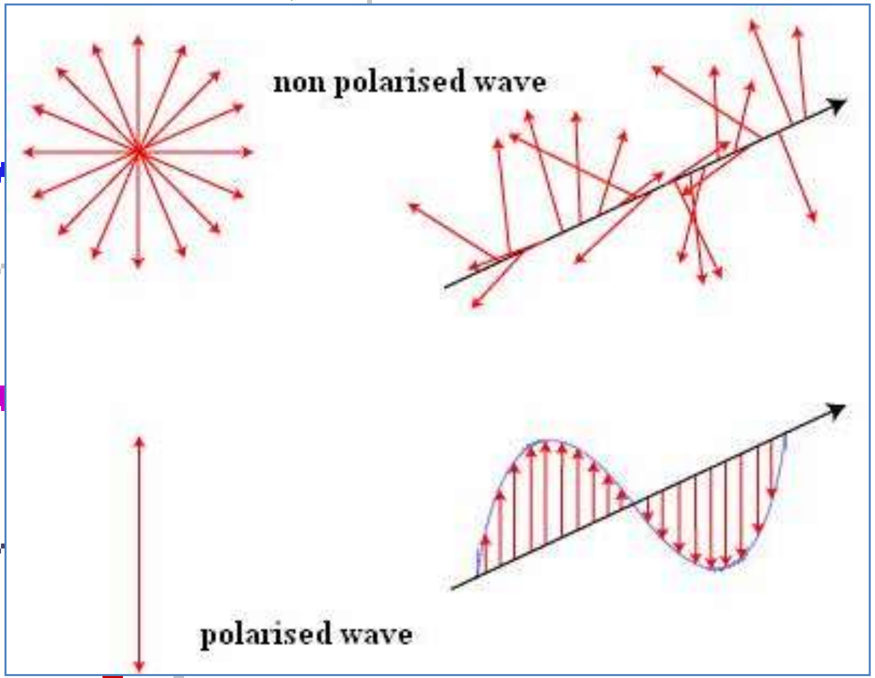
U.S. Flower beetle and it’s reflection. Upper frame image is taken with LCP filter, lower frame with a RCP filter.

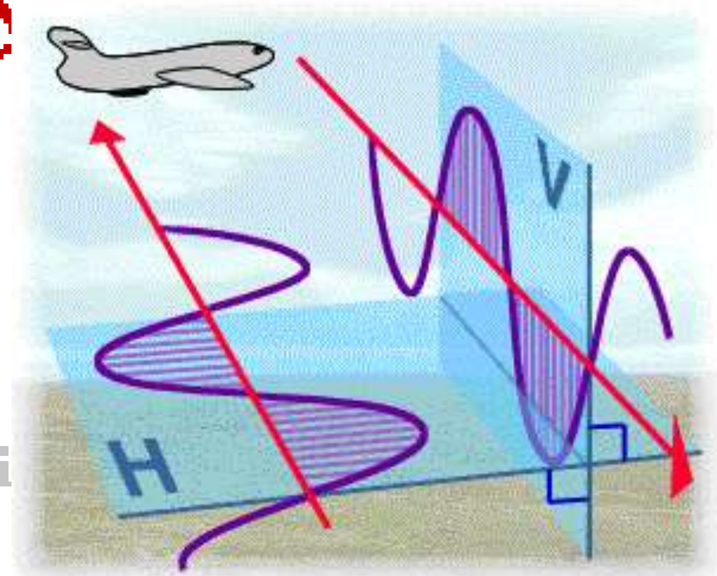

Electric Field
λ = wavelength
Magnetic Field
$\hat{x}$
velocity of light
ν = frequency (number of cycles per second)
$\hat{x}$
Animated Traveling Wave
4
2
0
-2
-4
-4
-3
-2
-1
1
2
3
4
x

$$\vec{E} = \hat{x}\, a \cos(\omega t - kz + \phi_a) + \hat{y}\, b \cos(\omega t - kz + \phi_b)$$

What if $\phi_b - \phi_a$ is Random ??







$$k = \omega\sqrt{\mu\,\varepsilon_c} = \omega\sqrt{\mu\left(\varepsilon - j\frac{\sigma}{\omega}\right)}$$

$$= \omega\sqrt{\mu\varepsilon}\left[\sqrt{1 - j\frac{\sigma}{\varepsilon\omega}}\right] = \beta - j\alpha$$

$$\tilde{E} = \hat{x}\,E_0\,e^{-\alpha z}e^{-j\beta z}$$

$$\vec{E} = \hat{x}E_o e^{-\alpha z}\cos(\omega t - \beta z)$$

Propagation Constant

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}$$

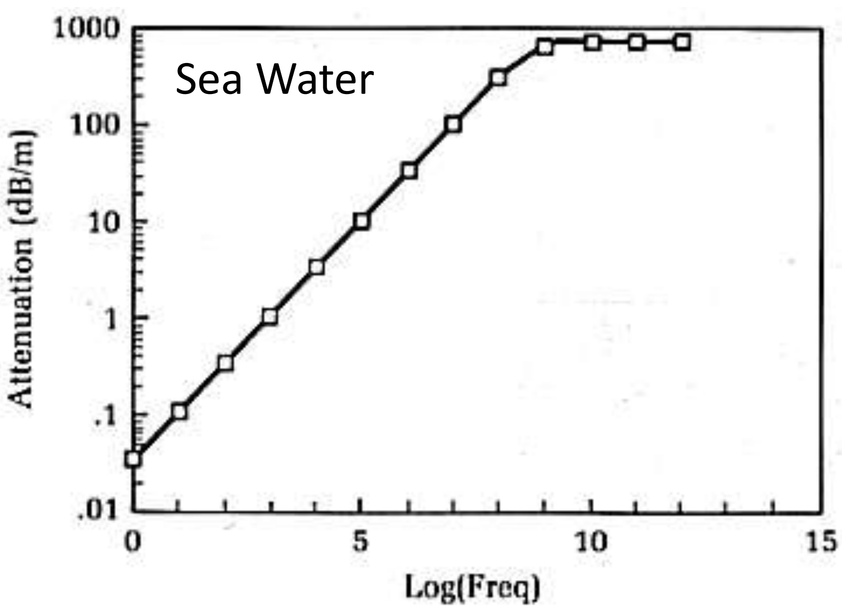
Loss Tangent:

$$\frac{\sigma}{\omega\varepsilon}\ldots \quad \text{if}\begin{cases} >> 1, \text{conductive} \\ << 1, \text{insulator(dielectric)} \end{cases}$$

$$\textit{with} \quad \varepsilon = \varepsilon_r\varepsilon_o$$

$$\alpha = \text{attenuation coefficient}$$

$$\frac{\omega}{\beta} = \text{velocity}$$



Material	Conductivity, σ(S/m)	Material	Conductivity, σ(S/m)
Silver	6.17×10^{7}	Fresh water	10^{-3}
Copper	5.80×10^{7}	Distilled water	2×10^{-4}
Gold	4.10×10^{7}	Dry soil	10^{-5}
Aluminum	3.54×10^{7}	Transformer oil	10^{-11}
Brass	1.57×10^{7}	Glass	10^{-12}
Bronze	10^{7}	Porcelain	2×10^{-13}
Iron	10^{7}	Rubber	10^{-15}
Seawater	4	Fused quartz	10^{-17}

Decibels

$$10\log_{10}\left(\frac{I}{I_o}\right)$$

Linear Medium

$\Rightarrow f = f$ ☺

$\sqrt{1+x} \simeq 1 + \frac{x}{2}$ for $x \ll 1$

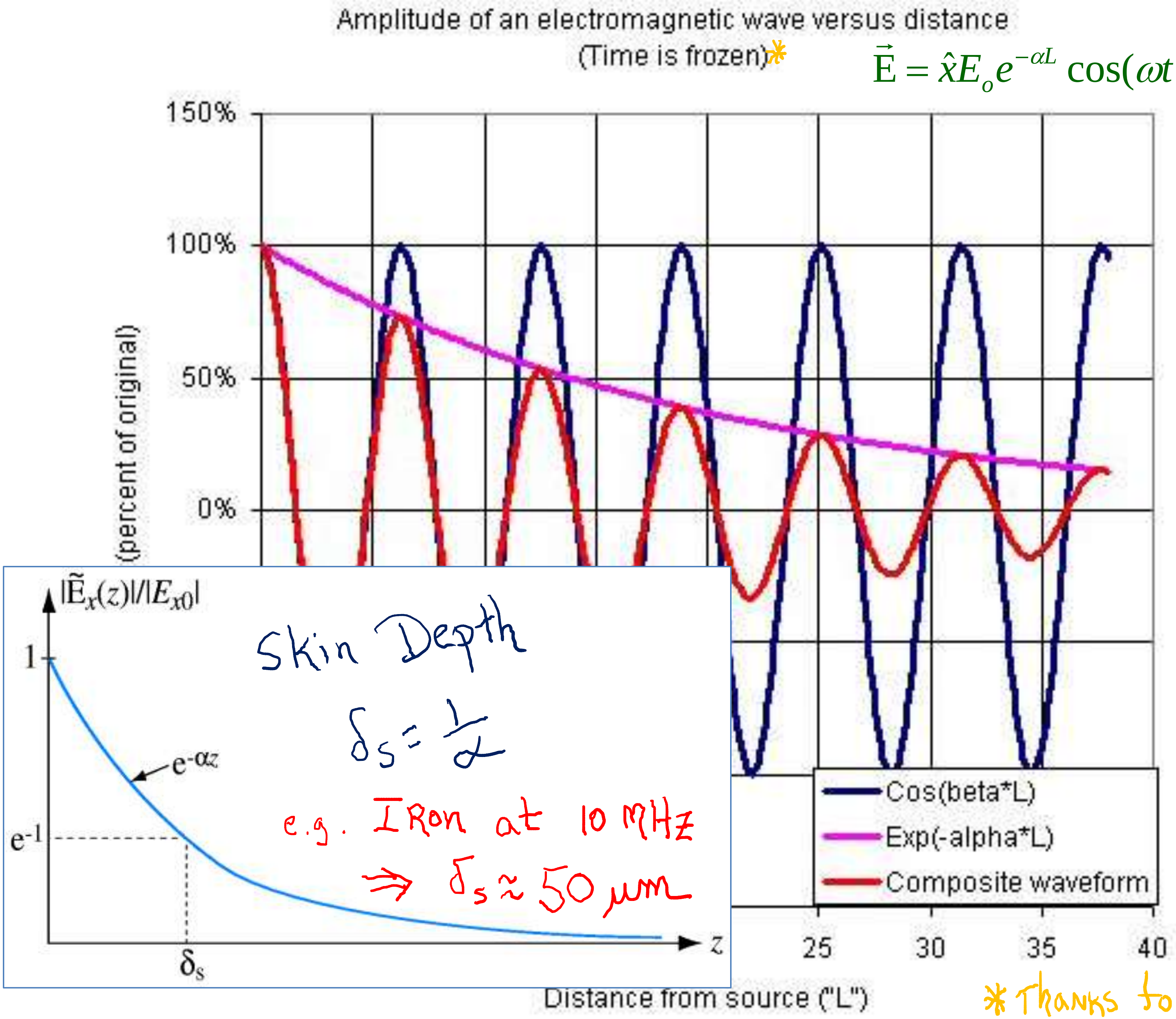
	Any Medium	Lossless Medium ($\sigma = 0$)	Low-loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega \left[\frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right] \right]^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f \mu \sigma}$ * same	(Np/m)
$\beta =$	$\omega \left[\frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1 \right] \right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f \mu \sigma}$	(rad/m) 45°
$\eta_c =$	$\sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	(Ω)
$u_p =$	ω/β	$1/\sqrt{\mu\varepsilon}$	$1/\sqrt{\mu\varepsilon}$	$\sqrt{4\pi f/\mu\sigma}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Notes: $\varepsilon' = \varepsilon$; $\varepsilon'' = \sigma/\omega$; in free space, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$; in practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' > 100$.

Loss tangent

$* \sqrt{\frac{\omega\mu\sigma}{2}}$

$$\vec{E} = \hat{x} E_o e^{-\alpha L} \cos(\omega t - \beta L)$$



* Thanks to Hiro

For each of the following combination of parameters, determine if the material is a low-loss dielectric, a quasi-conductor, or a good conductor, and then calculate α, β, λ, u_p, and η_c:

(a) glass with $\mu_r = 1$, $\varepsilon_r = 5$, and $\sigma = 10^{-12}$ S/m at 10 GHz,

(b) animal tissue with $\mu_r = 1$, $\varepsilon_r = 12$, and $\sigma = 0.3$ S/m at 100 MHz,

(c) wood with $\mu_r = 1$, $\varepsilon_r = 3$, and $\sigma = 10^{-4}$ S/m at 1 kHz.

	Case (a)	Case (b)	Case (c)
$\sigma/\omega\varepsilon$	3.6×10^{-13}	4.5	600
Type	low-loss dielectric	quasi-conductor	good conductor
α	8.42×10^{-11} Np/m	9.75 Np/m	6.3×10^{-4} Np/m
β	468.3 rad/m	12.16 rad/m	6.3×10^{-4} rad/m
λ	1.34 cm	51.69 cm	10 km
u_p	1.34×10^8 m/s	0.52×10^8 m/s	0.1×10^8 m/s
η_c	$\simeq 168.5\ \Omega$	$39.54 + j31.72\Omega$	$6.28(1+j)\ \Omega$

δ_s basically infinite 10 cm ~ 1 km

Based on wave attenuation and reflection measurements conducted at 1 MHz, it was determined that the intrinsic impedance of a certain medium is $28.1\angle 45^\circ$ (Ω) and the skin depth is 2 m. Determine (a) the conductivity of the material, (b) the wavelength in the medium, and (c) the phase velocity.

(a) Since the phase angle of η_c is 45°, the material is a good conductor. Hence,

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = 28.1e^{j45^\circ} = 28.1\cos 45^\circ + j28.1\sin 45^\circ,$$

or

$$\frac{\alpha}{\sigma} = 28.1\cos 45^\circ = 19.87.$$

Since $\alpha = 1/\delta_s = 1/2 = 0.5$ Np/m,

$$\sigma = \frac{\alpha}{19.87} = \frac{0.5}{19.87} = 2.52\times 10^{-2} \text{ S/m}.$$

(b) Since $\alpha = \beta$ for a good conductor, and $\alpha = 0.5$, it follows that $\beta = 0.5$. Therefore,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.5} = 4\pi = 12.57 \text{ m}.$$

(c) $u_p = f\lambda = 10^6 \times 12.57 = 1.26\times 10^7$ m/s.


Oregon State University
Conceptual Wave Park
1-2 miles offshore
Magnetic Shaft
anchored to
sea floor
Electric Coil
secured to
heaving buoy
Permanent Magnet
Linear Generator Buoy

But 1st ...

what do ϵ_0 and μ_0 actually mean? $\Rightarrow$

$$\vec{E} = \hat{R}\,\frac{q}{4\pi\epsilon_0 R^2} \qquad \vec{F} = q'\vec{E}$$

different q

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi} \qquad \vec{F} = q'\,\vec{u}\times\vec{B}$$

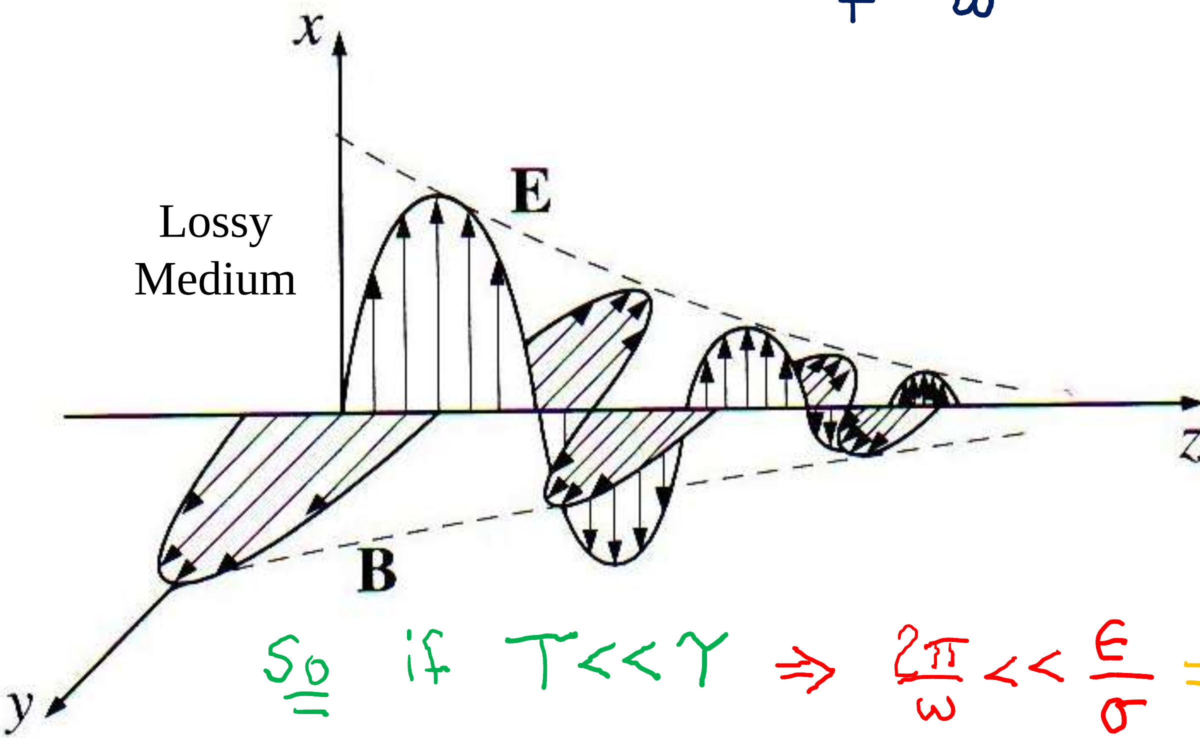
T = period of wave $= \frac{1}{f} = \frac{2\pi}{\omega}$

$$\rho_v = \rho_{v_0}\, e^{-t/\tau_r}$$

with

$$\tau = \epsilon/\sigma$$

Relaxation time constant



So if $T \ll \tau \Rightarrow \frac{2\pi}{\omega} \ll \frac{\epsilon}{\sigma} \Rightarrow \underbrace{2\pi\left(\frac{\sigma}{\omega\epsilon}\right) \ll 1}_{\text{Low-Loss}}$

Waves (by their nature) transmit energy….

- In the time domain, the power density **and** direction of the E & M energy flow is given by:

$$\vec{S} = \vec{E} \times \vec{H} \qquad \mathrm{W/m^2}$$

$\vec{S}$ is the Poynting Vector

$P = \int_A \vec{S} \cdot \hat{n} \, dA$

- like waves against the shore, $\vec{S}$ surges E & M power with period T/2…

∴ we look for the time average

$$\langle \vec{S} \rangle = \langle \vec{E} \times \vec{H} \rangle = \frac{1}{2} \mathrm{Re}\{\tilde{E} \times \tilde{H}^*\}$$

Time Domain

Phasors

analagous to $\frac{1}{2} \mathrm{Re}\{\tilde{V} \tilde{I}^*\}$ from 210

Cool!

$$= \frac{|E_0|^2}{2\eta} \hat{z} \ \mathrm{W/m^2}$$

$\eta = \sqrt{\frac{\mu}{\epsilon}}$

Not just for uniform plane waves!

where the phasors are defined as:

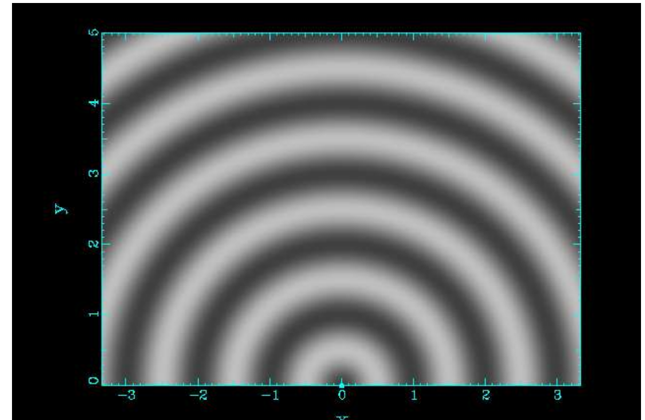
$$\tilde{E} = \hat{x} E_0 e^{-jkz} \quad \& \quad \tilde{H} = \hat{y} \left(\frac{E_0}{\eta} \right) e^{-jkz}$$

what is η?

Example:

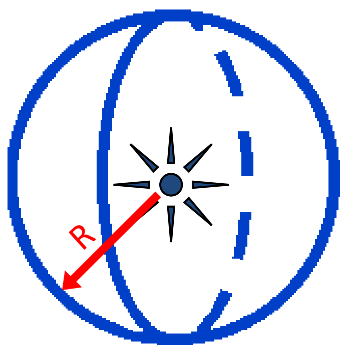
Assume a light bulb gives off light @ a given wavelength (500 nm). The light radiates equally in all directions. It is a 100% efficient 100 W lamp.

- What is $|E_0|$ @ a distance 10m from the lamp?



Spherical phase fronts

Isotropic Radiator



Note: R^2 factor very important!

Distribute radiated power over surface area of sphere $= 4\pi R^2$

What assumptions have we made here???

$$\left|\vec{S}\right| = \frac{|E_0|^2}{2\eta_0} = \frac{100\text{W}}{4\pi \times 10^2\,\text{m}^2} \quad \Rightarrow \quad |E_0| \cong 7.75\,\frac{\text{V}}{\text{m}}$$

Assume air or vacuum; $\eta_0 \approx 377\Omega$

The magnetic field of a plane wave traveling in air is given by $\mathbf{H} = \hat{\mathbf{x}}\, 50 \sin(2\pi \times 10^7 t - ky)$ (mA/m). Determine the average power density carried by the wave.

$$\mathbf{H} = \hat{\mathbf{x}} 50 \sin(2\pi \times 10^7 t - ky) \quad \text{(mA/m)},$$

$$\mathbf{E} = -\eta_0 \hat{\mathbf{y}} \times \mathbf{H} = \hat{\mathbf{z}} \eta_0 50 \sin(2\pi \times 10^7 t - ky) \quad \text{(mV/m)},$$

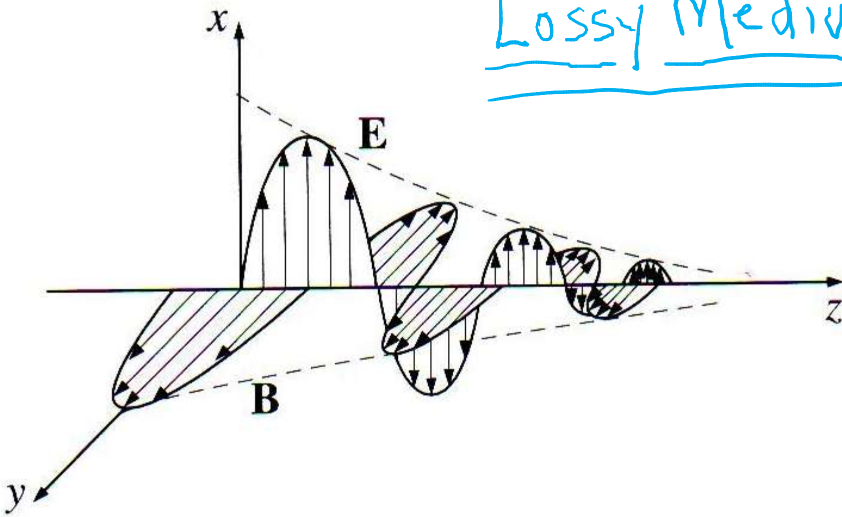
$$\mathbf{S}_{\text{av}} = (\hat{\mathbf{z}} \times \hat{\mathbf{x}}) \frac{\eta_0 (50)^2}{2} \times 10^{-6} = \hat{\mathbf{y}} \frac{120\pi}{2} (50)^2 \times 10^{-6} = \hat{\mathbf{y}} 0.48 \quad (\text{W/m}^2).$$

Q: Is $\vec{E} = \hat{x} E_o \cos(\omega t^2 - \beta z)$ a viable electromagnetic wave??

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$

wave equation as derived from Maxwell's

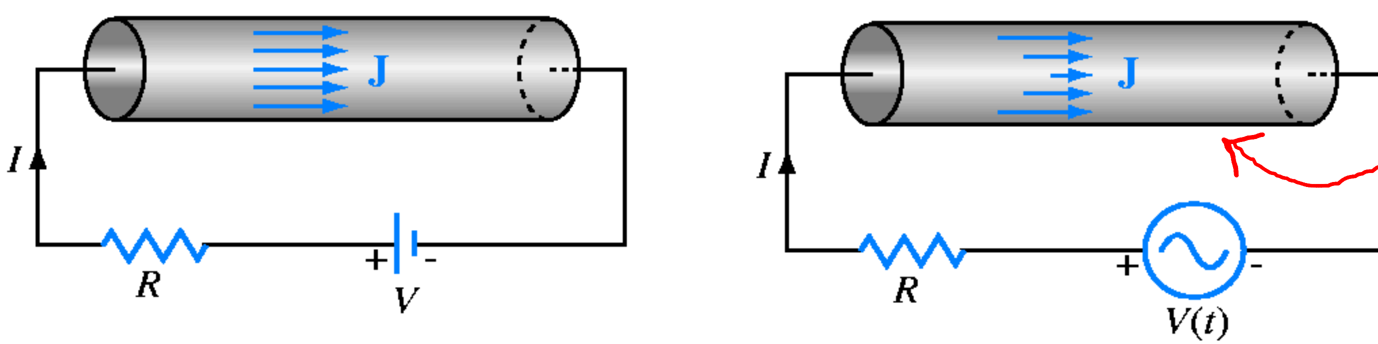
Lossy Medium



$|\vec{E}| \propto e^{-\alpha z}$

Power Density $\propto |\vec{E}|^2 \propto e^{-2\alpha z}$

Pretty Darned Good Conductors



Check out where the current is!

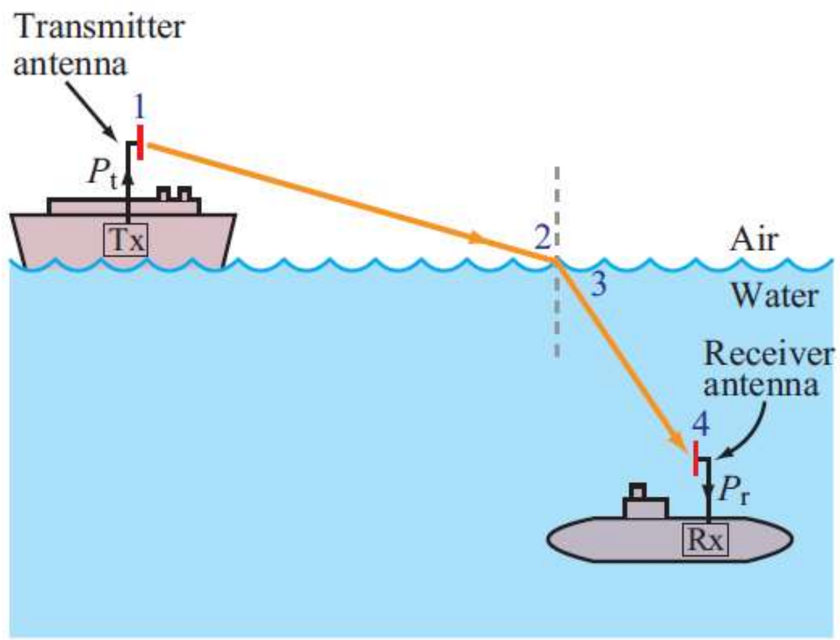
skin depth $\Rightarrow$ $\delta_L \approx \dfrac{1}{\sqrt{\pi f \mu \sigma}}$ e.g. $\delta_{copper} \sim 20\,\mu m$ at 10 MHz

Example 7-6: Power Received by a Submarine Antenna

A submarine at a depth of 200 m below the sea surface uses a wire antenna to receive signal transmissions at 1 kHz. Determine the power density incident upon the submarine antenna due to the EM wave of Example 7-4.

Solution: From Example 7-4, $|\tilde{E}(0)| = |E_{x0}| = 4.44$ (mV/m), $\alpha = 0.126$ (Np/m), and $\eta_c = 0.044\angle 45^\circ$ (Ω). Application of Eq. (7.109) gives

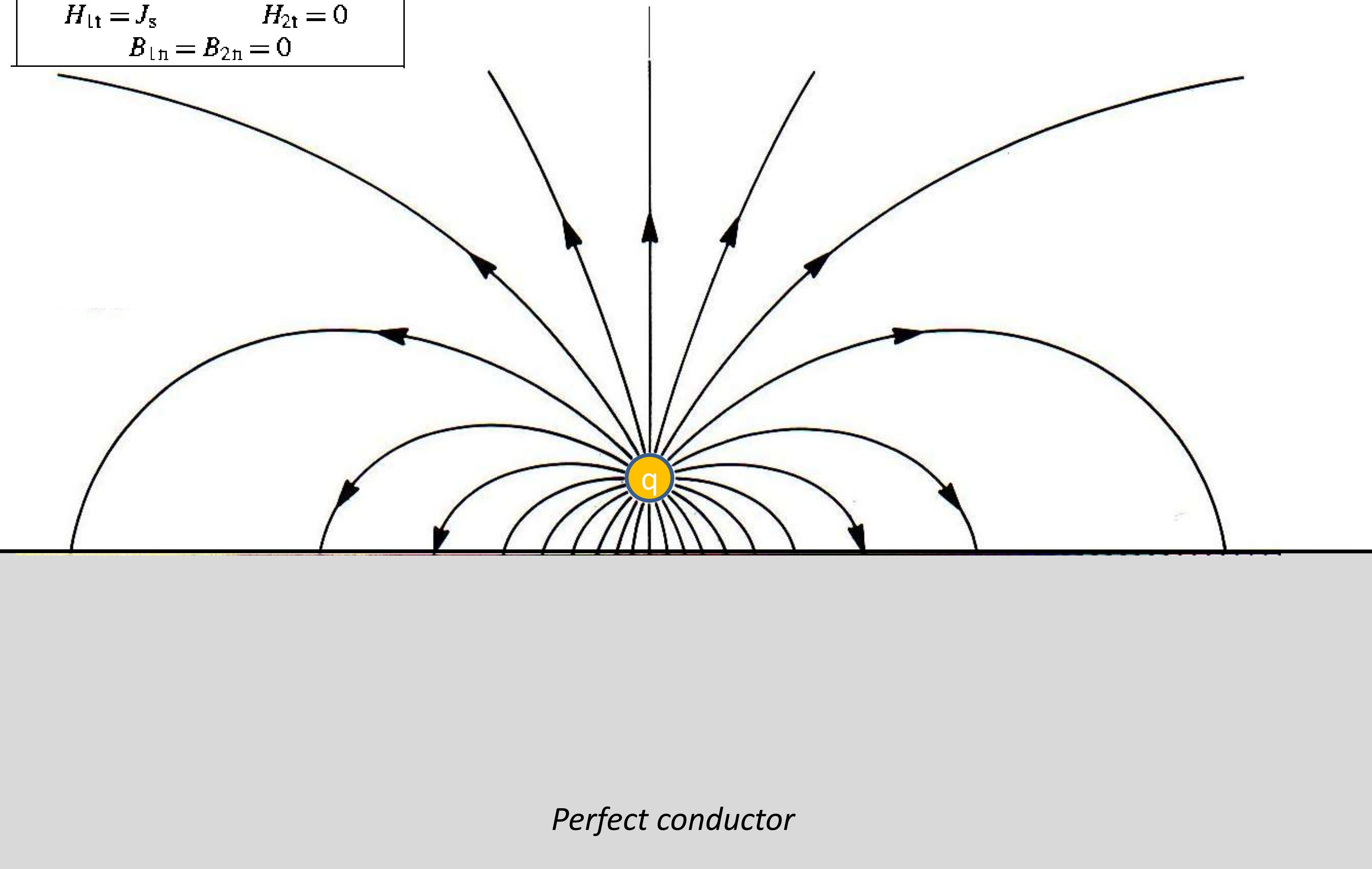
$$\begin{aligned}\mathbf{S}_{av}(z) &= \hat{\mathbf{z}}\,\frac{|E_0|^2}{2|\eta_c|}\,e^{-2\alpha z}\cos\theta_\eta \\ &= \hat{\mathbf{z}}\,\frac{(4.44\times 10^{-3})^2}{2\times 0.044}\,e^{-0.252z}\cos 45^\circ \\ &= \hat{\mathbf{z}}\,0.16e^{-0.252z} \qquad (\text{mW/m}^2).\end{aligned}$$



At $z = 200$ m, the incident power density is

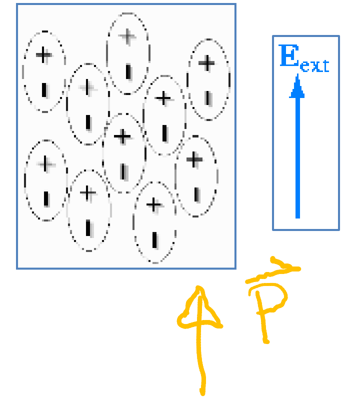
$$\begin{aligned}\mathbf{S}_{av} &= \hat{\mathbf{z}}\,(0.16\times 10^{-3}e^{-0.252\times 200}) \\ &= 2.1\times 10^{-26} \qquad (\text{W/m}^2).\end{aligned}$$

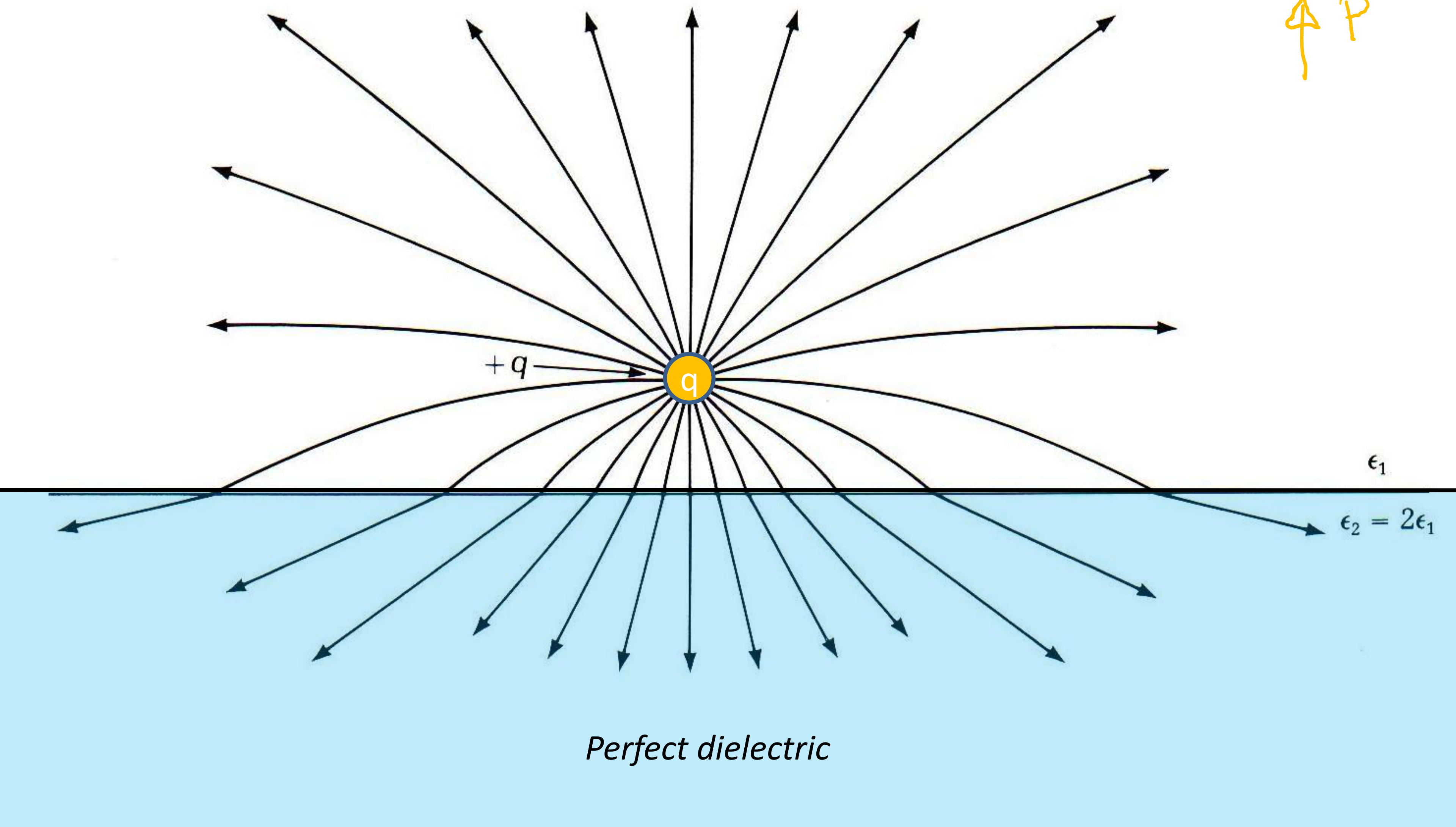
Medium 1 Dielectric	Medium 2 Conductor
$E_{1t} = E_{2t} = 0$	
$D_{1n} = \rho_s$	$D_{2n} = 0$
$H_{1t} = J_s$	$H_{2t} = 0$
$B_{1n} = B_{2n} = 0$	



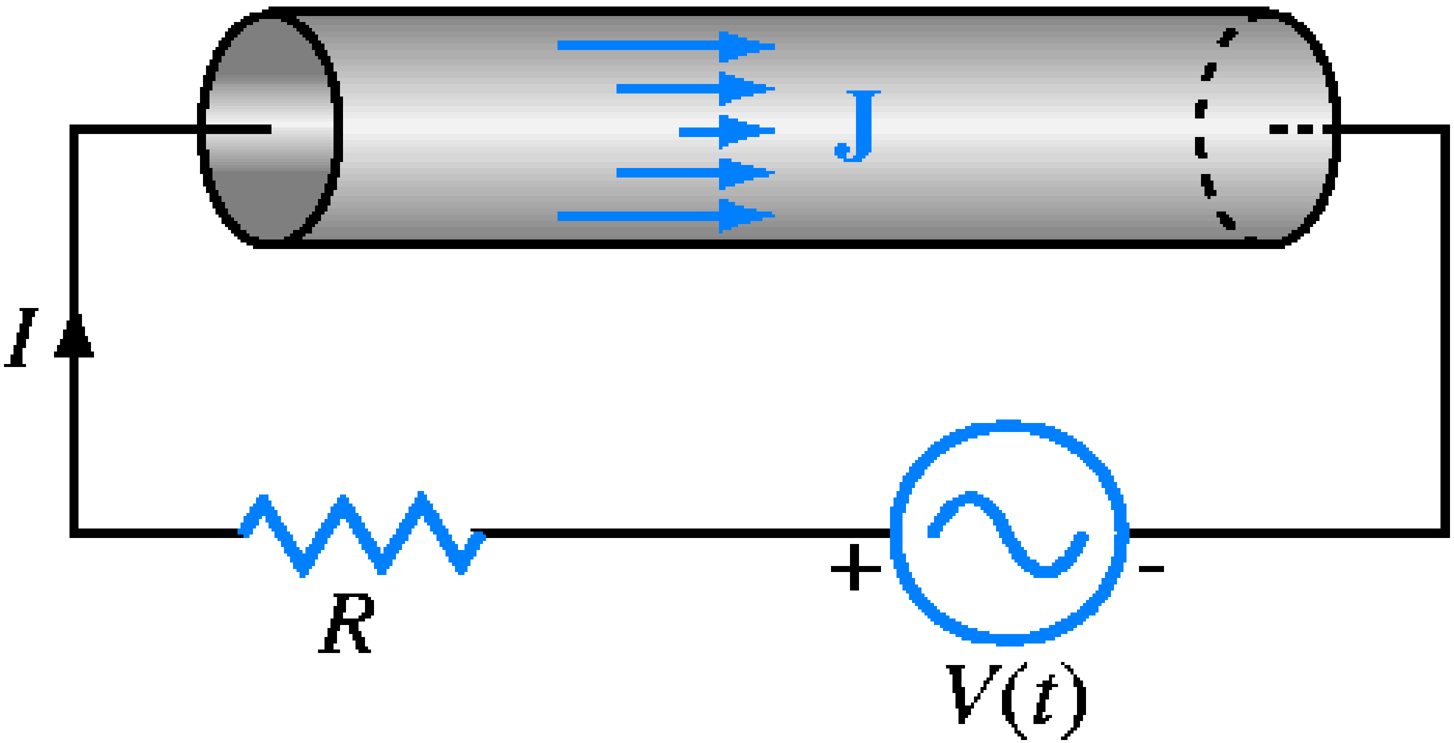
Medium 1 Dielectric	Medium 2 Dielectric
$E_{1t} = E_{2t}$	
$D_{1n} - D_{2n} = \rho_s$	
$H_{1t} = H_{2t}$	
$B_{1n} = B_{2n}$	

$E_{2n} = \dfrac{\epsilon_1}{\epsilon_2} E_{1n}$ for $\rho_s = 0$

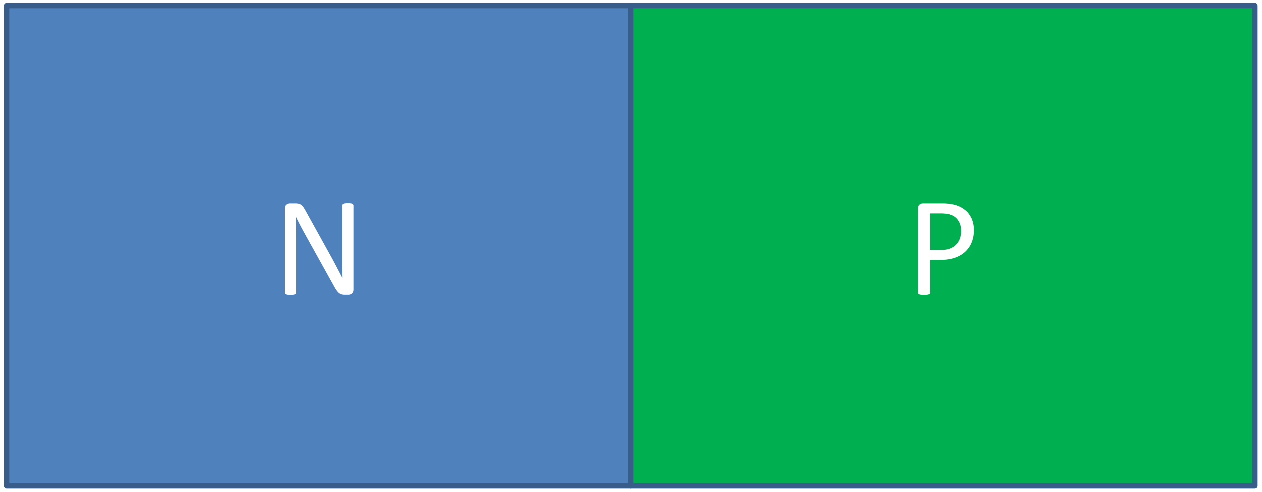




Even at relatively low frequencies used for high power transmission (50–60 Hz), non-uniform distribution of current still occurs in sufficiently thick conductors. For example, the skin depth of a copper conductor is approximately 8.57 mm at 60 Hz, so high current conductors are usually hollow to reduce their mass and cost.



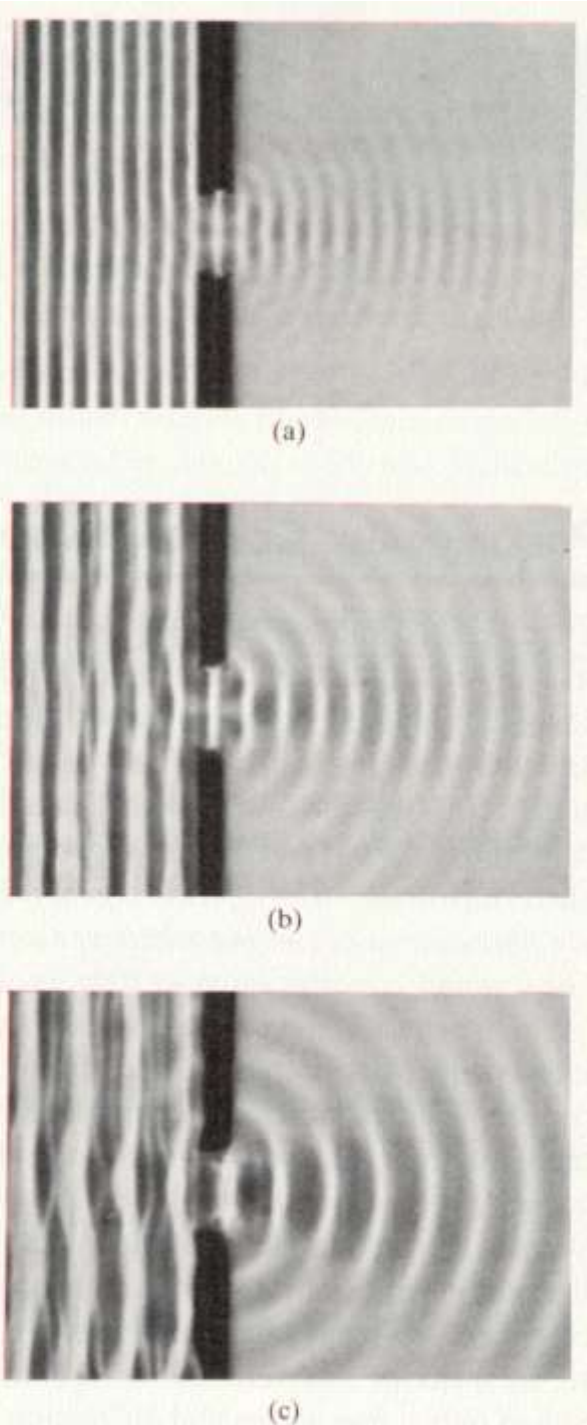
Since the current tends to flow in the periphery of conductors, the effective cross-section of the conductor is reduced. This increases the effective *AC* resistance of the conductor, since resistance is inversely proportional to the cross-sectional area in which the current actually flows. The AC resistance often is many times higher than the *DC* resistance, causing a much higher energy loss due to ohmic heating (also called I^2R loss).



$$E_{1t} = E_{2t}$$

$$D_{1n} - D_{2n} = \rho_s$$

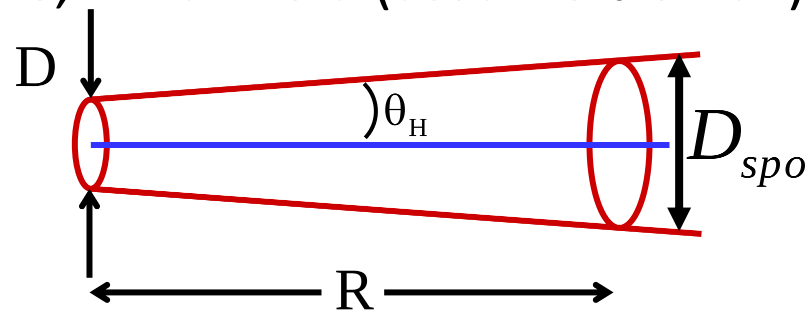
$$J_{1n} = J_{2n}$$



Circular Aperture, in Far field (assume θ small)

$$\sin\theta_H \cong \frac{1.22\lambda}{D}$$

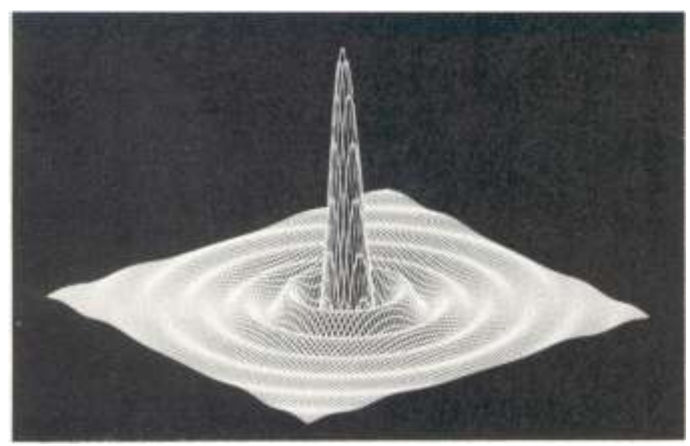
rad



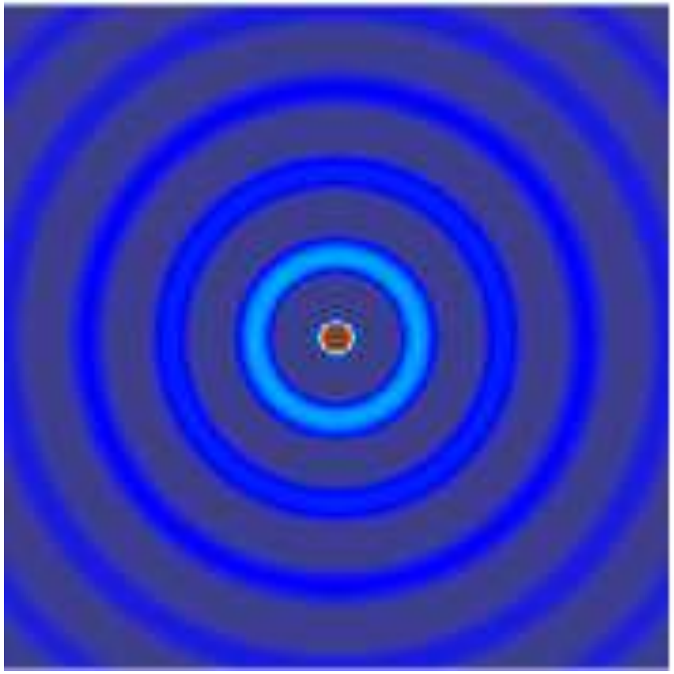
spot size, if $\theta << 1$

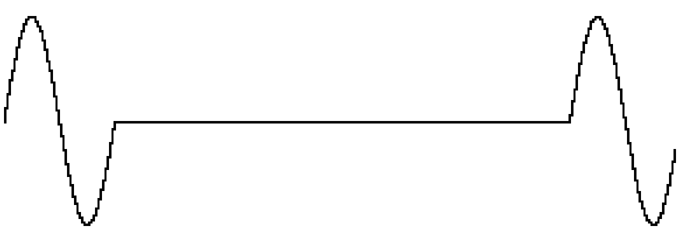
$$D_{spot} \approx \frac{2.44\lambda}{D} \times R$$

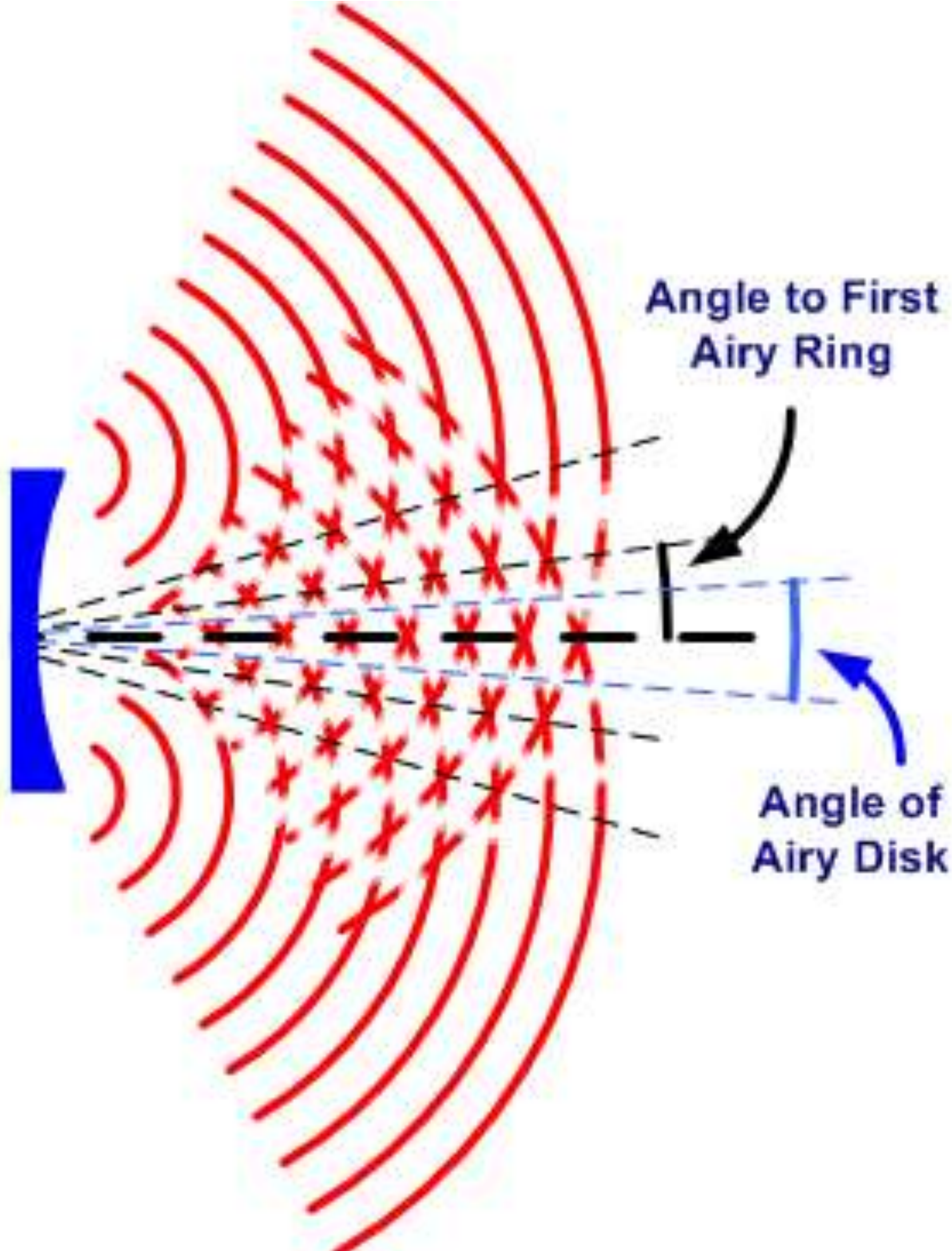
What about "near field"?

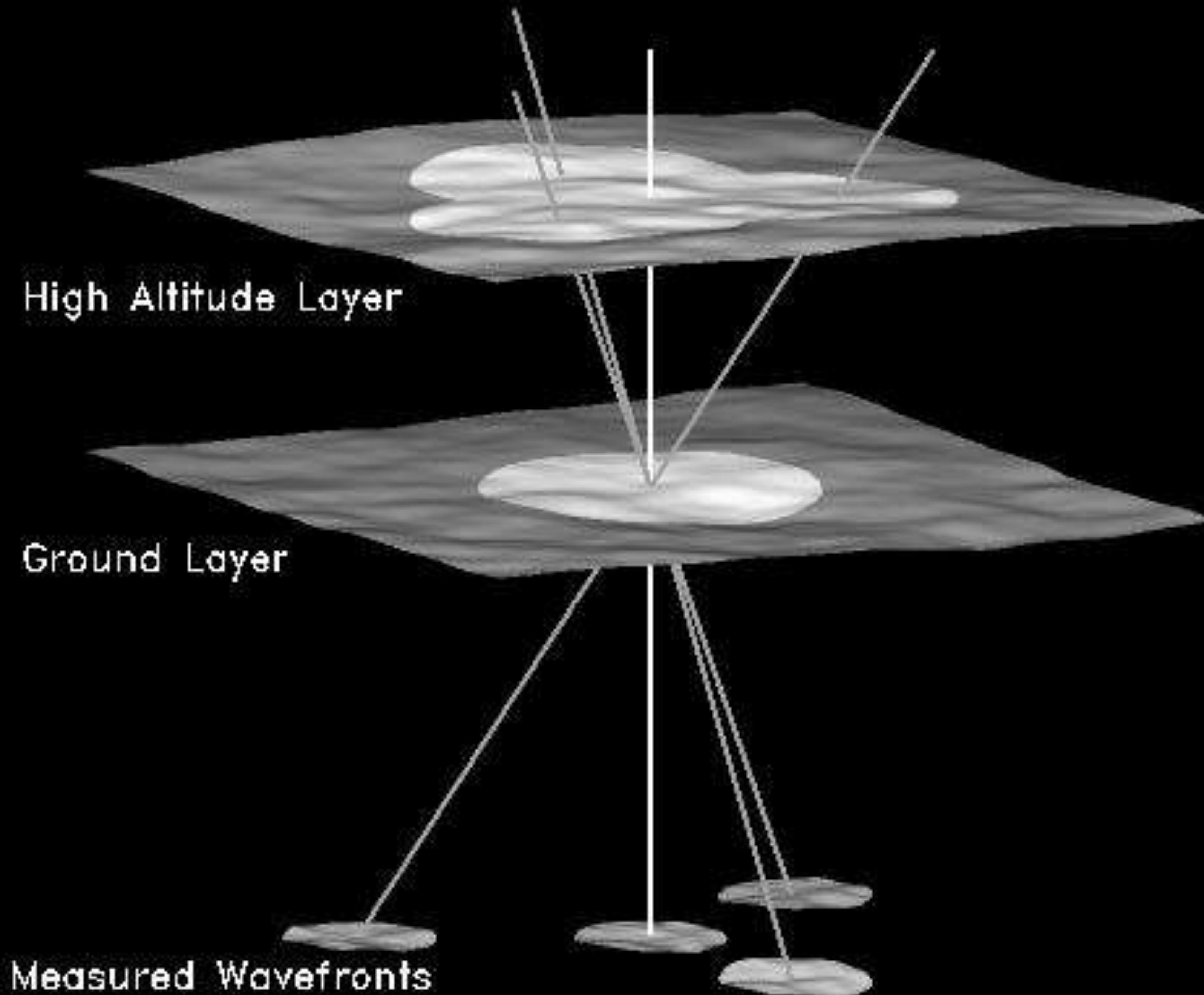


Electric field distribution in the far-field is proportional to the Fourier Transform of the transmitter/receiver distribution.





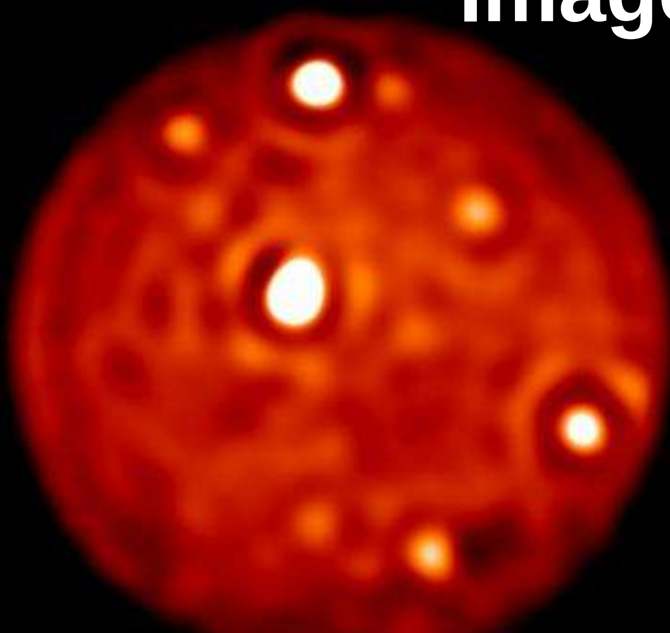

Angle to First
Airy Ring
Angle of
Airy Disk


High Altitude Layer
Ground Layer
Measured Wavefronts

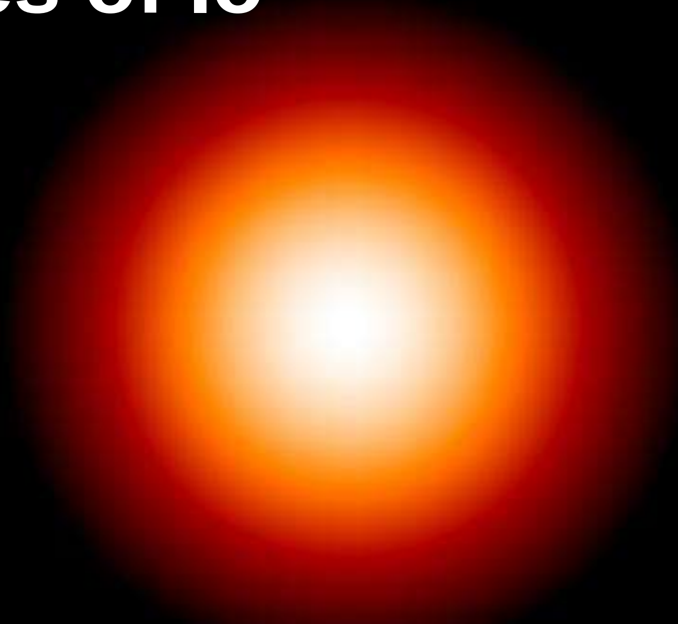
Images of Io

2.2 microns AO Keck

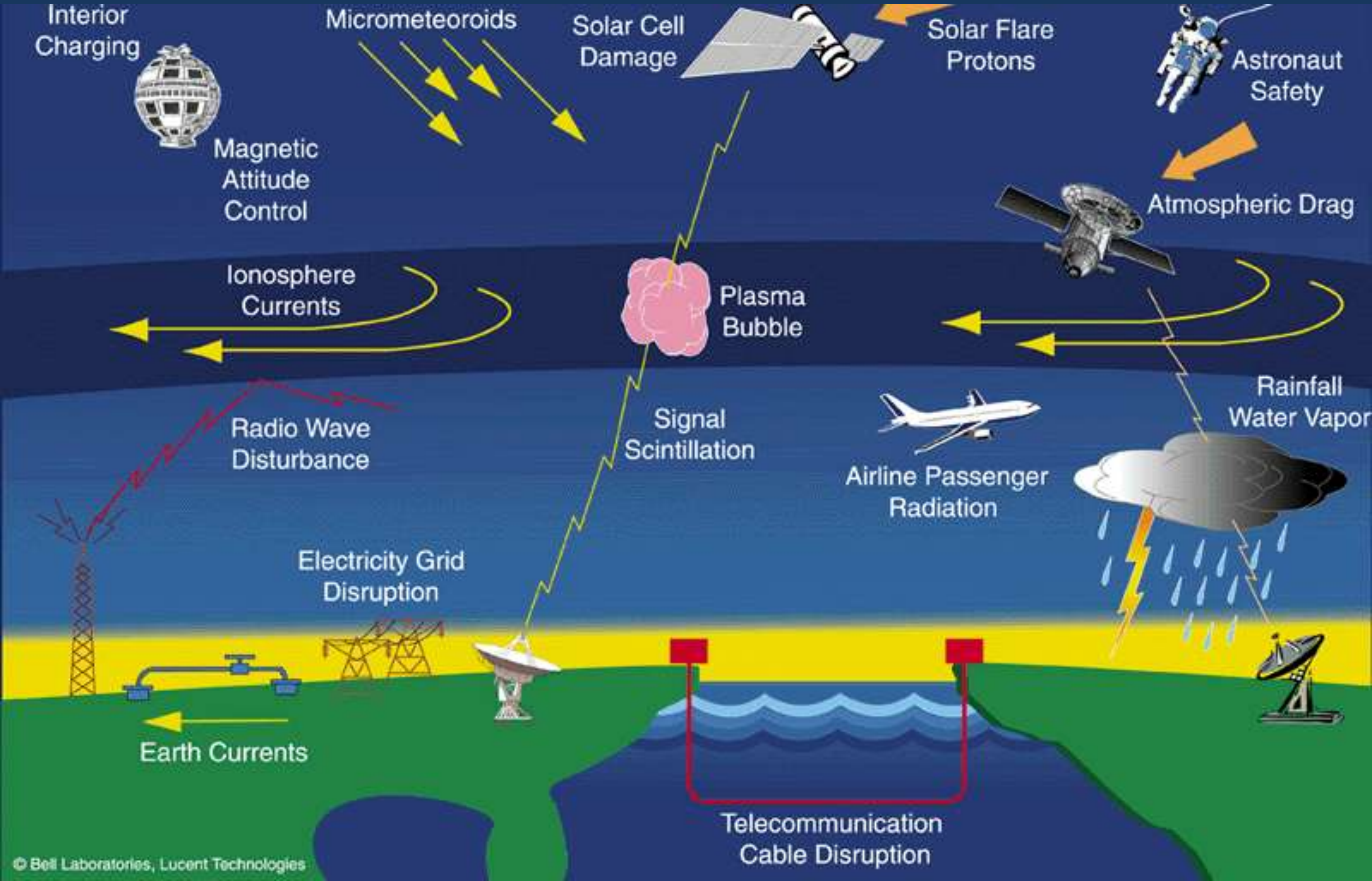
Galileo Visible

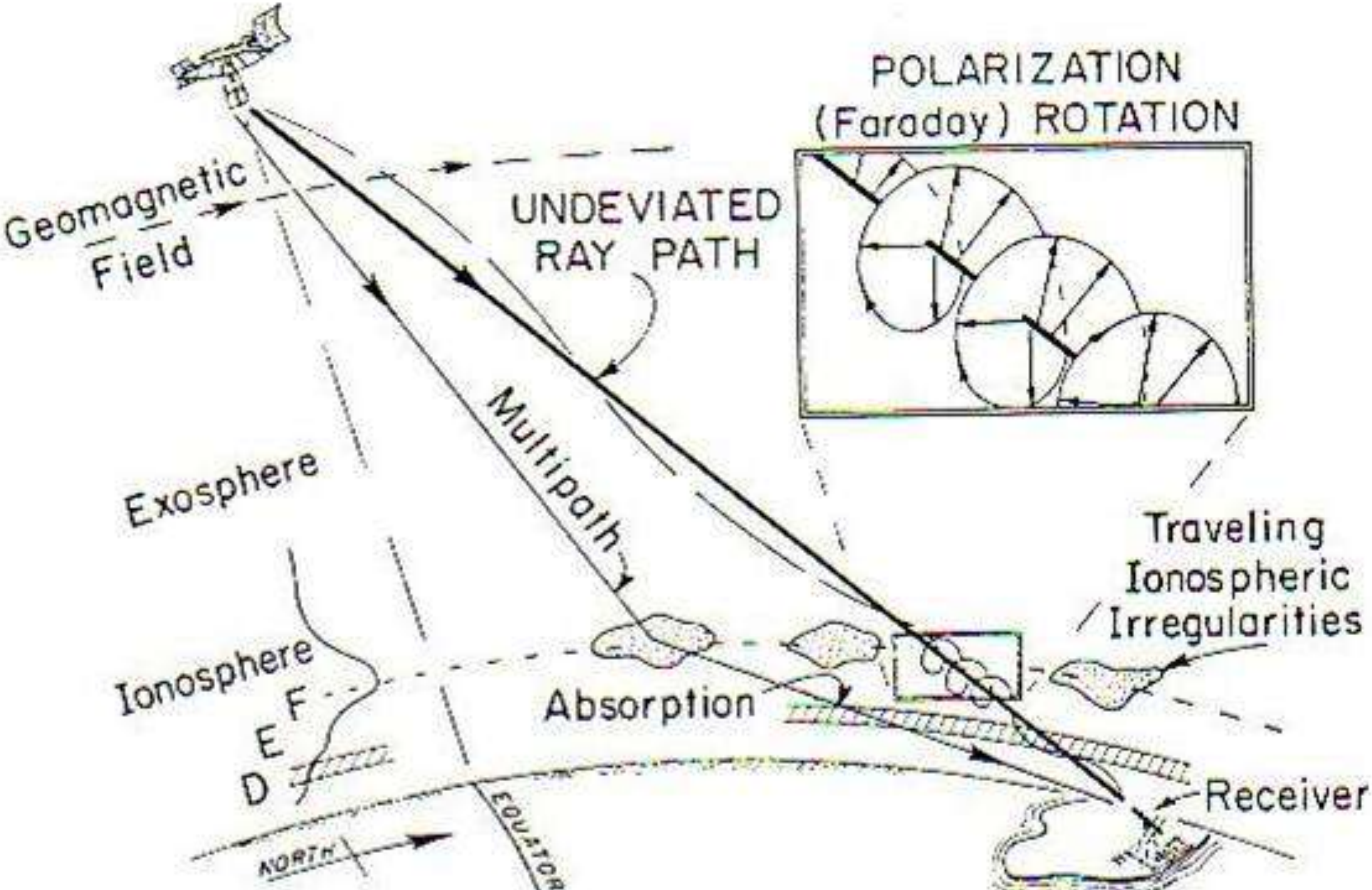


3.5 microns AO Keck



Keck, no AO


Interior Charging
Micrometeoroids
Solar Cell Damage
Solar Flare Protons
Astronaut Safety
Magnetic Attitude Control
Atmospheric Drag
Ionosphere Currents
Plasma Bubble
Rainfall Water Vapor
Radio Wave Disturbance
Signal Scintillation
Airline Passenger Radiation
Electricity Grid Disruption
Earth Currents
Telecommunication Cable Disruption
© Bell Laboratories, Lucent Technologies



At 433 MHz (UHF), one should expect in the order of 1.5 complete rotations of the wavefront as it transits the ionosphere, whereas at 1.2 GHz less than a quarter of one rotation is likely.

♦ Question: What generates an E & M wave (UPW or otherwise)?

Answer: An accelerated electron….

(a moving electron is not enough)

1) e^- (at rest) $\Rightarrow$ $\vec{E}$, but no $\vec{H}$

2) e^- (constant velocity) $\Rightarrow$ $\vec{E}$, $\vec{H}$ constant (roughly)

3) e^- (acceleration) $\Rightarrow$ $\vec{E}$ & $\vec{H}$ time-varying (what Maxwell needs)

Index of Refraction

$$N \equiv c\sqrt{\varepsilon\mu} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

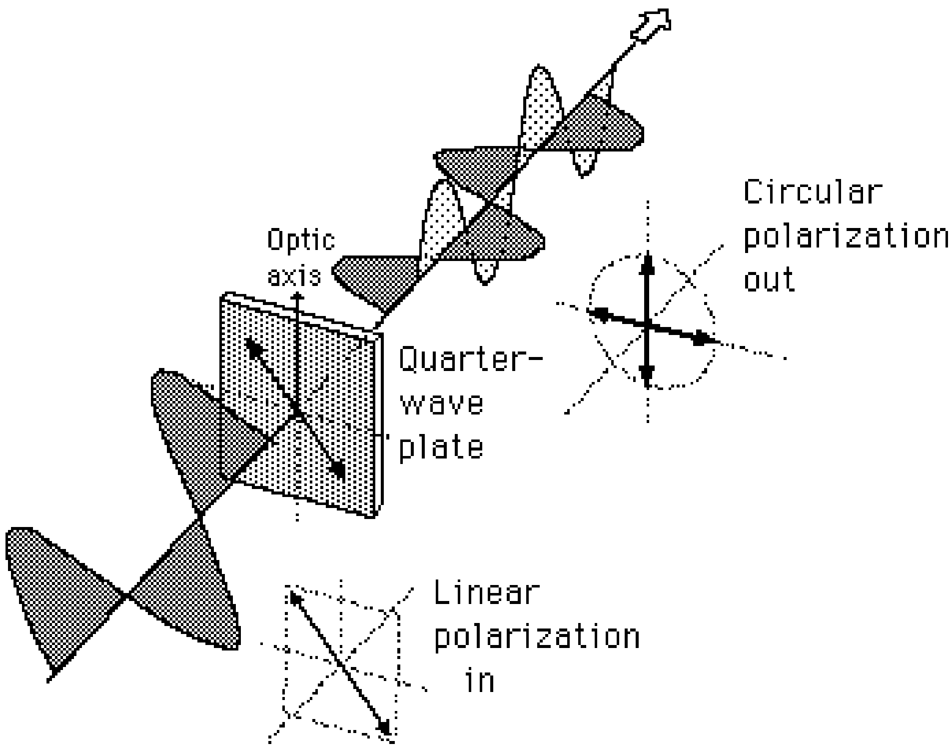
$$= n + j\,k$$

if $\quad \mu = \mu_0$

then $\;N = \sqrt{\varepsilon_c}$

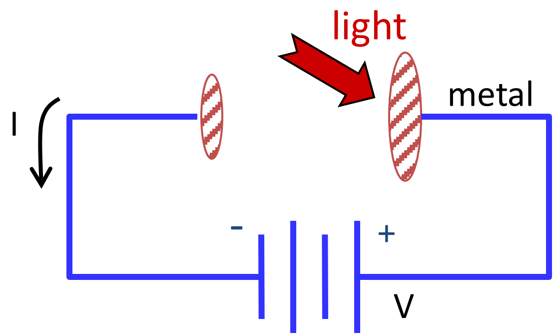
* Complex Refractive Index

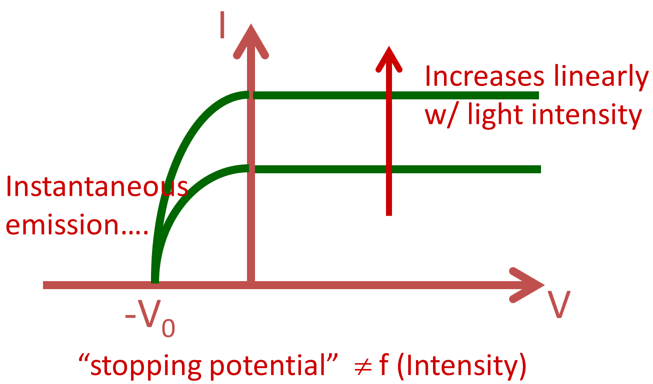
('n' affects properties; 'k' absorbs)



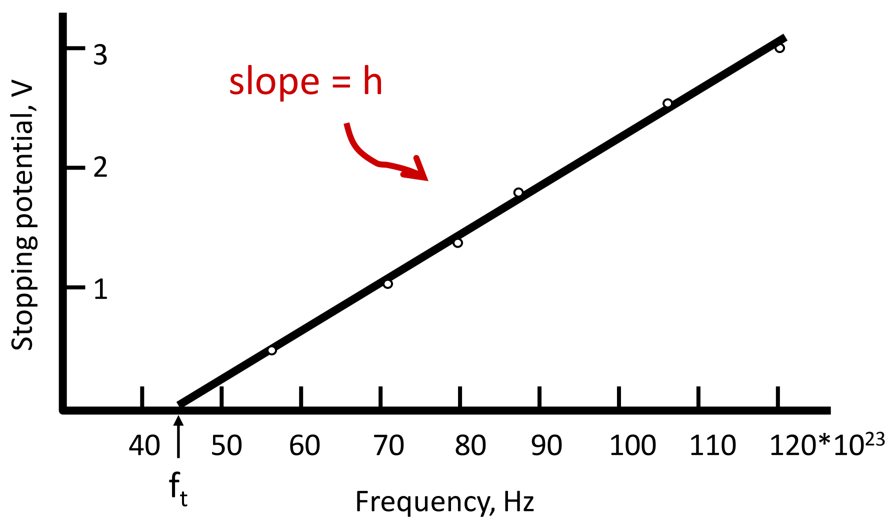
E & M particles??

Hertz
(Exp.)





Photoelectric
Einstein
(theory)



Milliken Data
(confirms Einstein)

h = Plank’s constant
= 6.626×10^{-34} J•s

Photons!! $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{1240\ \text{eV}}{\lambda\ \text{nm}}$

500 nm => 2.5 eV
= 4×10^{-19} J

10 GHz => 3 x 10^7 nm
=> 0.0000413 eV

- But what about waves?
- How big is a photon?
- What's the E-field strength of a photon?
- Can a photon be polarized ??

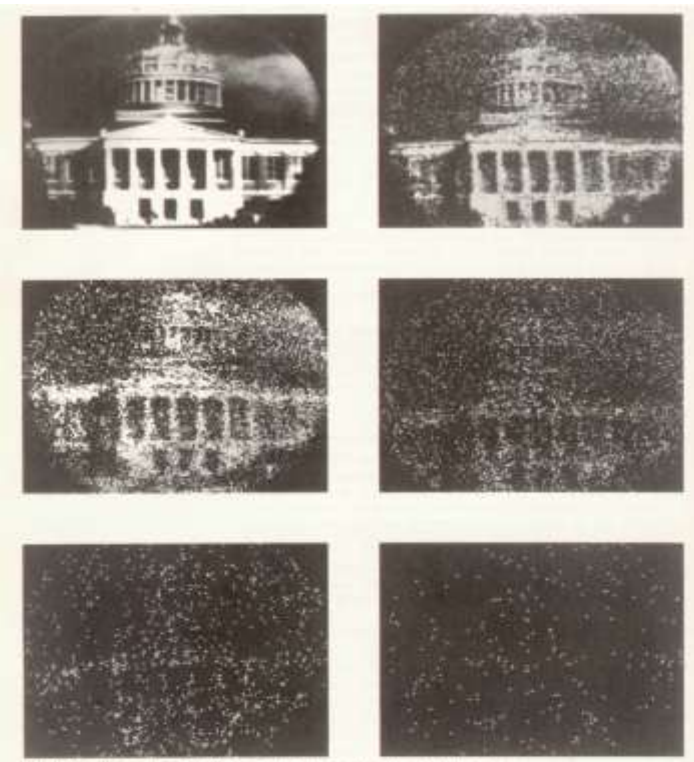


Figure 3-8. In these low-light-level images, a bright spot shows the arrival position for a single photon. The number of photons used to record the images from the upper left to the lower right were 10^7, 64×10^3, 16×10^3, 4×10^3, 10^3, and 250. (Courtesy of M. Morris, University of Rochester.)

<u>Sample numbers of mean photon flux (photons/s•m²)</u>

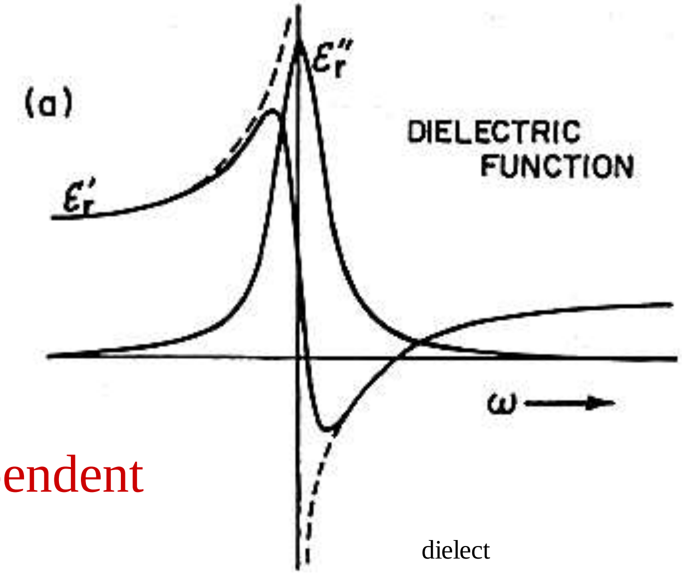
Laser	10^{26}
Bright Sunlight	10^{18}
Starlight	10^{10}

<u>Note</u>
it's ~ 4×10^{11} mm to the Moon !

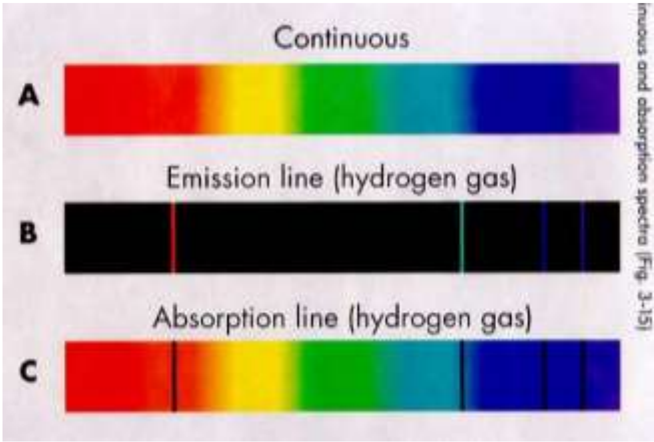
Note: we can (and do!) actually "photon-count" with some sensors!

Absorption

$$\varepsilon = \varepsilon' + j\varepsilon''$$



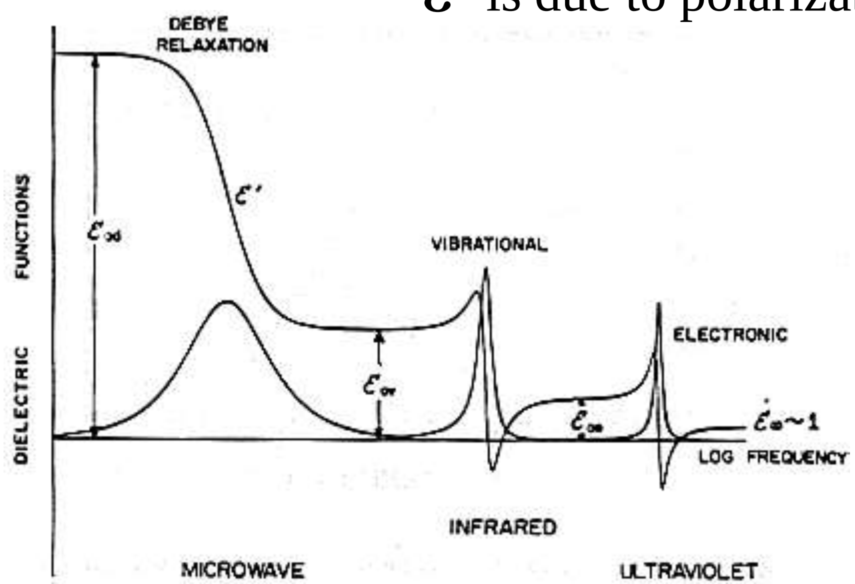
dielect



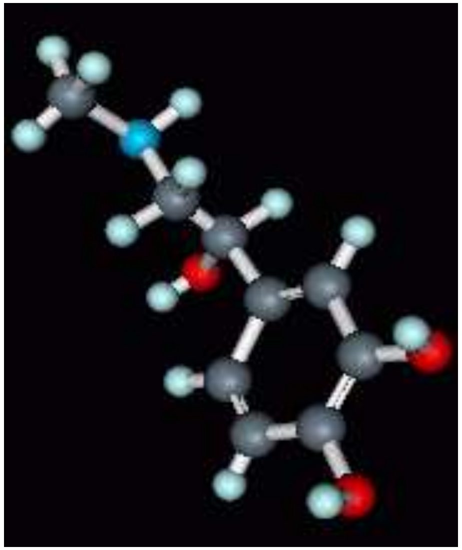
♦ Are ε' & ε'' independent of each other?

ε'' has to do with attenuation

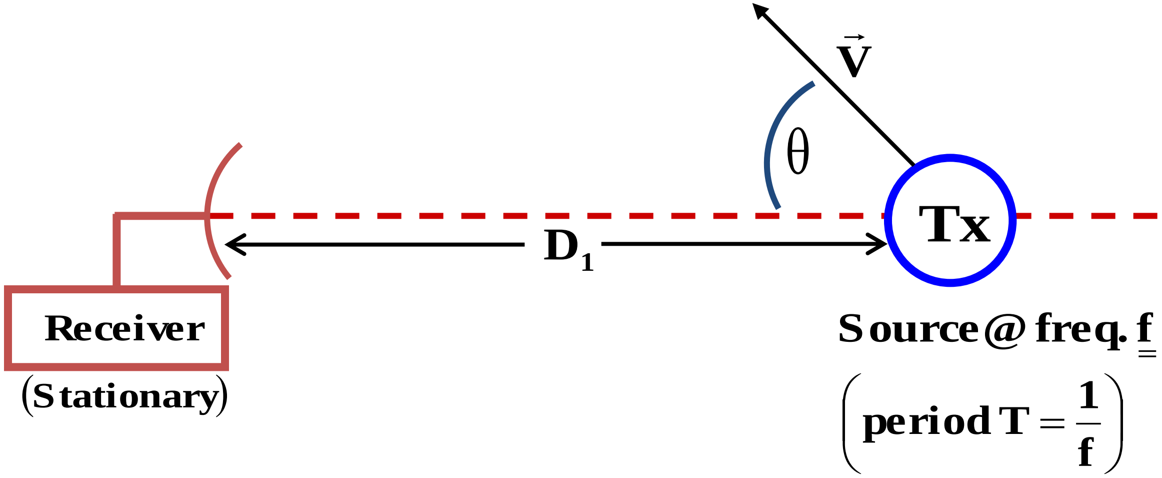
ε' is due to polarizability



debye



♦ What does $\varepsilon'_{\infty} = 1$ mean to you?

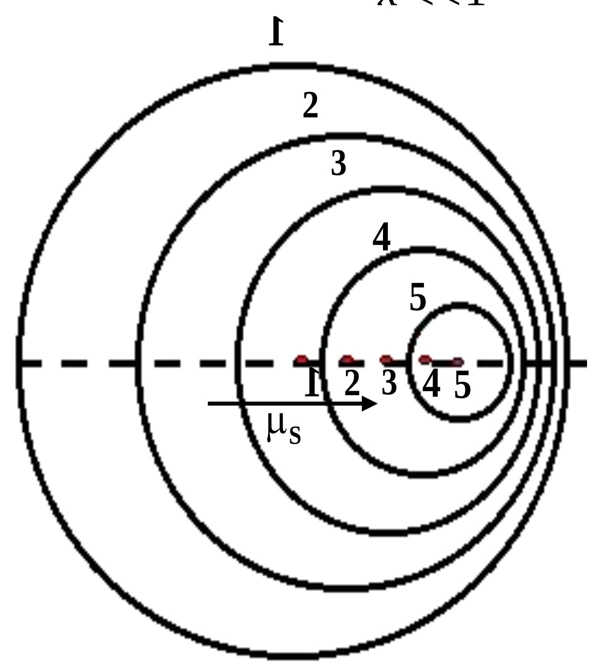


$$\sqrt{1+x} \approx 1 + \frac{x}{2} \quad \text{if} \quad x << 1$$

$$\frac{1}{1-x} \approx 1 + x$$

$$x << 1$$

$$f_{obs} \approx f\left(1 + \frac{v}{c}\cos\theta\right)$$



So,

$$\Delta f = f_{obs} - f$$

$$\underbrace{\approx \left(\frac{v}{c}\cos\theta\right) f}_{\text{Doppler Shift}}$$